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CONTENT ARTICLE IN ECONOMICS

Macroeconomic Stabilization When the Natural Real Interest Rate Is Falling

Sebastien Buttet and Udayan Roy

The authors modify the Dynamic Aggregate Demand-Dynamic Aggregate Supply model in Mankiw's widely used intermediate macroeconomics textbook to discuss monetary policy when the natural real interest rate is falling over time. Their results highlight a new role for the central bank's inflation target as a tool of macroeconomic stabilization. They show that even when the zero lower bound is *not* binding, a prudent central bank must match every decrease in the natural real interest rate with an equal increase in the target rate of inflation in order to stabilize the risk of the economy falling into a deflationary spiral, which is an acute case of simultaneously falling output and inflation in which the economy's self-correcting forces are inactive.

Keywords *intermediate macroeconomics, natural interest rate, secular stagnation, zero lower bound*

JEL codes *E12, E52, E58*

Recent discussions of secular stagnation have drawn attention to the steady decrease in the natural real interest rate in the United States since the 1980s.¹ In this article, we investigate how a nation's central bank should respond to decreases in the natural real interest rate when the zero lower bound (ZLB) on nominal interest rates is potentially binding. In particular, if output is at the full-employment level and inflation is at the central bank's inflation target, should the central bank feel free to ignore decreases in the natural real interest rate? Our answer is no. We argue that decreases in the natural real interest rate increase an economy's vulnerability to the deflationary spiral, which is an acute case of simultaneously falling output and inflation in which the economy's self-correcting forces become inactive. We also show that a central bank can neutralize this danger by simply raising its inflation target.

Intermediate macroeconomics textbooks do not discuss what policymakers should do in response to changes in the natural real interest rate.² We hope to show that a standard textbook

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model and standard graphical techniques can be used to present this new aspect of macroeconomic stabilization to students. We work out the comparative static effects of a fall in the natural real interest rate in the Dynamic Aggregate Demand-Aggregate Supply (DAD-DAS) model of short-run macroeconomic dynamics in Mankiw (2013, ch. 15), a widely used intermediate macroeconomics textbook, modified, as in Buttet and Roy (2014), to formally include the ZLB on the nominal interest rate.

In Mankiw's DAD-DAS model, equilibrium output and inflation are determined at the intersection of two curves: a negatively sloped dynamic aggregate demand curve and a positively sloped dynamic aggregate supply curve. Irrespective of the equilibrium levels of inflation and output at a given date, the economy converges over time to the model's unique long-run equilibrium, in which output is at the full-employment level and inflation is at the central bank's target rate. However, when the explicit requirement that the nominal interest rate set by the central bank must be non-negative is added, Buttet and Roy (2014) show that the familiar negatively sloped aggregate demand curve becomes a kinked curve with a negatively sloped segment (when the ZLB is non-binding) and a positively sloped segment (when the ZLB is binding).³ This leads to two (rather than one) long-run equilibria: (1) the stable equilibrium discussed above and (2) an unstable equilibrium at which the ZLB is binding and the slightest shock can set off a deflationary spiral.⁴

The equilibrium inflation rate at any given date turns out to be the crucial determinant of the economy's subsequent destiny. If inflation at a given date is higher than the negative of the natural real interest rate, then all is well: the economy converges to the stable long-run equilibrium. If, on the other hand, inflation at a given date drops below the negative of the natural real interest rate, the economy enters a deflationary spiral. Inflation's *danger level* is equal to the negative of the natural real interest rate; inflation must be kept above this danger level at all costs.

Consider an economy that is at the stable long-run equilibrium, with inflation equal to the central bank's target inflation rate and higher than the danger level. An unfavorable demand shock (or a favorable inflation shock) could reduce inflation from the central bank's target inflation rate to below the danger level, thereby initiating a deflationary spiral. This is why the job of the central bank in our model is no longer restricted to the standard one of achieving full employment and low inflation. The central bank must also do what it can to reduce the risk of a shock-induced deflationary spiral. This is why a decrease in the natural real interest rate makes macroeconomic stabilization harder. When the natural real interest rate decreases, the negative of it increases, raising the danger level of inflation closer to the central bank's target rate. Consequently, the possibility that a shock would cause inflation to fall from the central bank's target rate (in the stable equilibrium) to below the danger level becomes more likely.⁵

Luckily, the very nature of the danger points to a way of neutralizing it. If the risk of a deflationary spiral has increased because the danger level of inflation has increased and comes closer to the target rate, an obvious solution is to raise the target rate as well. We show that any increase in the economy's vulnerability to the deflationary spiral can be neutralized by an increase in the central bank's inflation target.⁶ If the natural real interest rate decreases by a certain amount and if the central bank responds by increasing the target inflation by the same amount, then the gap between the inflation target and the negative of the natural real interest rate would remain unaffected, and therefore the chance that a shock of a given size would precipitate a deflationary spiral would stay unchanged.⁷

While it is fairly obvious that when the ZLB is binding, a central bank that wishes to maintain full employment must increase its target inflation rate in lockstep with decreases in the natural

real interest rate, the analysis outlined above shows that even when the ZLB is *not* binding, a prudent central bank must match every decrease in the natural real interest rate with an equal increase in the target inflation rate in order to keep the economy's vulnerability to the deflationary spiral from increasing.⁸ Intermediate macroeconomics textbooks do not address this issue. We hope to show that a standard textbook model and standard graphical techniques can be used to present this new aspect of macroeconomic stabilization to students.

Note that our focus on the natural real interest rate is related to the current research and debate on *secular stagnation* (e.g., Eggertsson and Mehrotra 2014). In an address to the National Association for Business Economics that is often cited in discussions of secular stagnation, Larry Summers referred to "changes in the structure of the economy that have led to a significant shift in the natural balance between savings and investment, causing a decline in the equilibrium or normal real rate of interest that is associated with full employment" (Summers 2014b, 69).⁹ However, economists do not agree on how to define secular stagnation, what causes it, or whether it exists at all (Eichengreen 2014).¹⁰ Our article shows how macroeconomic stabilization is affected by secular stagnation as defined by Summers.

The rest of the article is organized as follows. The next section reviews historical data about real interest rates and the output gap for the United States in the last thirty years, and proposes a simple way to estimate the natural real interest rate for use in intermediate macroeconomics courses. In the following section, we present the DAD-DAS model of Mankiw (2013) modified to include the ZLB constraint. We derive the kinked demand curve and characterize the model's long-run equilibria and study their stability. In the subsequent two sections, we show how decreases in the natural real interest rate increase the economy's vulnerability to a deflationary spiral and how raising the central bank's target inflation can insulate the economy from this danger. We conclude by discussing some issues not formally modeled in this article.

MOTIVATING EVIDENCE

A salient feature of bond markets, both in the United States and abroad, is that real interest rates and the natural real interest rate have been in a downward trend for the past thirty years (King and Low 2014; Barsky, Justiniano, and Melosi 2014; Laubach and Williams 2003).¹¹ While real interest rates can be easily inferred from inflation and nominal yield data, the natural real interest rate is a theoretical construct that is not directly observable and must be estimated.¹² Here, we propose a model, simple enough to be included in intermediate macroeconomics courses, to estimate the natural real interest rate. We run a linear regression between real interest rates, r_t , and the output gap, ΔY_t ¹³:

$$r_t = \beta_0 + \beta_1 \Delta Y_t + \varepsilon_t \quad (1)$$

where the disturbance term ε_t is independent and identically distributed (i.i.d.) over time. We interpret the estimate of the intercept β_0 as an estimate of the natural real interest rate because β_0 estimates the real interest rate when output gap is zero.

We present estimates of β_0 and β_1 for each decade between 1981 and 2013 in table 1. While our model is much simpler than Laubach and Williams (2003), we also find that the natural real interest rate has been falling since the 1980s and has become negative after the financial crisis.¹⁴

TABLE 1
The Natural Real Interest Rate, 1981–2013 Quarterly Data (Std Error in Parentheses)

Decade	β_0	β_1	R^2	N
1981–1990	0.0507 (0.0041)	$-8.55^{10^{-5}}$ ($1.8^{10^{-5}}$)	.36	40
1991–2000	0.0366 (0.0013)	$-2.16^{10^{-6}}$ ($5.9^{10^{-6}}$)	.26	40
2001–2008	0.0184 (0.0030)	$-1.21^{10^{-5}}$ ($1.4^{10^{-5}}$)	.03	32
2009–2013	-0.0118 (0.0104)	$-2.11^{10^{-5}}$ ($1.2^{10^{-5}}$)	.16	20
All: 1981–2013	0.0240 (0.0021)	$-6.88^{10^{-6}}$ ($6.5^{10^{-6}}$)	.06	132

In the next section, we present a theoretical macroeconomic model of the business cycle where the ZLB is potentially binding. Later, we use this model to analyze the impact of a decline in the natural interest rate on inflation and output.

A MODEL OF MACROECONOMIC DYNAMICS

As we saw in the previous section, the natural real interest rate has been decreasing in the United States since the 1980s. We wish to show that a decrease in the natural real interest rate has important implications for macroeconomic stabilization and that these implications can be easily discussed in undergraduate intermediate macroeconomics courses. To that end, we analyze in the next section the comparative static effects of a decrease in the natural real interest rate using a modified version of Mankiw's DAD-DAS model that incorporates the ZLB on the nominal interest rate. That modified DAD-DAS model (discussed in Buttet and Roy 2014) is summarized in this section.

The Kinked *DAD* Curve

The DAD curve of the DAD-DAS model is shown in figure 1. Aggregate demand is inversely related to inflation when the ZLB is nonbinding but directly related to inflation when the ZLB is binding. To understand the kinked shape of the DAD curve, we must look at the equations that drive Mankiw's DAD-DAS model. *Goods market equilibrium* in period t is given by

$$Y_t = \bar{Y}_t - \alpha \cdot (r_t - \rho) + \varepsilon_t. \quad (2)$$

Here, \bar{Y}_t denotes the natural or long-run level of output, r_t is the real interest rate, ρ is the natural or long-run real interest rate, α is a positive parameter representing the responsiveness of aggregate expenditure to the real interest rate, and ε_t represents demand shocks. Under fiscal stimulus (an increase in government expenditure or a decrease in taxes), ε_t is positive; under fiscal austerity, ε_t is negative.¹⁵

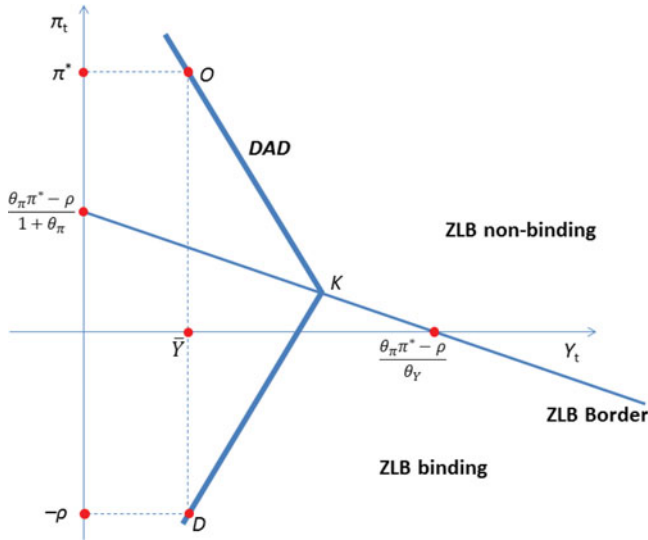


FIGURE 1 The Kinked DAD Curve. It is assumed that $\varepsilon_t = 0$ and $\bar{Y}_t = \bar{Y}$ for all t . The negatively sloped segment of the DAD curve satisfies equation (7), the positively sloped segment satisfies equation (8), and the ZLB border satisfies equation (6)

The *ex ante* real interest rate in period t is determined by the *Fisher equation* and is equal to the nominal interest rate i_t minus the inflation expected for the next period:

$$r_t = i_t - E_t \pi_{t+1}. \tag{3}$$

The expected inflation in the above equation is assumed to follow *adaptive expectations*:

$$E_t \pi_{t+1} = \pi_t. \tag{4}$$

The nominal interest rate in the Fisher equation (3) is assumed to be set by the central bank according to its *monetary policy rule*. This rule is $i_t = \pi_t + \rho + \theta_\pi \cdot (\pi_t - \pi^*) + \theta_Y \cdot (Y_t - \bar{Y}_t)$, where π^* is the central bank's *inflation target*, and the parameters θ_π and θ_Y are non-negative. The nominal interest rate must obey the ZLB (i.e., it must be non-negative). Therefore, the generalized monetary policy rule is

$$i_t = \max\{0, \pi_t + \rho + \theta_\pi \cdot (\pi_t - \pi^*) + \theta_Y \cdot (Y_t - \bar{Y}_t)\}. \tag{5}$$

Figure 1 shows the border that separates the (Y_t, π_t) -outcomes for which the ZLB is *not* binding from the (Y_t, π_t) -outcomes for which the ZLB is *binding*. Algebraically, the monetary policy rule (5) implies that the *ZLB border* satisfies

$$\pi_t + \rho + \theta_\pi \cdot (\pi_t - \pi^*) + \theta_Y \cdot (Y_t - \bar{Y}_t) = 0. \tag{6}$$

Above this border, the ZLB is not binding, and the nominal interest rate set by the central bank is positive ($i_t = \pi_t + \rho + \theta_\pi \cdot (\pi_t - \pi^*) + \theta_Y \cdot (Y_t - \bar{Y}_t) > 0$). On the border, the central bank chooses a zero interest rate but does so willingly, and not because it wanted a negative rate but could not choose it. Below the border, the ZLB is binding ($i_t = 0$).

Using equations (2) through (5), it is straightforward to show, following Buttet and Roy (2014), that

$$Y_t = \bar{Y}_t - \frac{\alpha\theta_\pi}{1 + \alpha\theta_Y}(\pi_t - \pi^*) + \frac{1}{1 + \alpha\theta_Y}\varepsilon_t \quad (7)$$

when the ZLB is nonbinding and

$$Y_t = \bar{Y}_t + \alpha \cdot (\pi_t + \rho) + \varepsilon_t. \quad (8)$$

when the ZLB is binding.

Equation (7) is graphed in figure 1 as the negatively sloped segment of the DAD curve above the ZLB border, which is where the ZLB is not binding. Equation (8) is graphed as the positively sloped segment of the DAD curve below the ZLB border, which is where the ZLB is binding. In this way, the introduction of the ZLB on the nominal interest rate yields a kinked DAD curve.

The positively sloped segment is meant to capture the idea that falling inflation is a special nightmare at the ZLB. As $i_t = 0$, any decline in current inflation ($\pi_t \downarrow$) means an increase in the current real interest rate ($r_t = i_t - E_t\pi_{t+1} = i_t - \pi_t = 0 - \pi_t = -\pi_t \uparrow$). The rising real interest rate reduces aggregate demand and output ($Y_t \downarrow$), as the familiar IS curve (2) dictates.¹⁶

The kinked DAD curve in figure 1 has been drawn assuming that the demand shock is absent ($\varepsilon_t = 0$). Consequently, $\pi_t = \pi^*$ and $Y_t = \bar{Y}_t$ satisfy equation (7). This is point O in the figure. Similarly, it is straightforward to check that $\pi_t = -\rho$ and $Y_t = \bar{Y}_t$ satisfy equation (8). This is point D in the figure. Outcomes O and D will play an important role in our discussion of the model's equilibrium below.

The DAS Curve

Coming now to aggregate supply, inflation, π_t , is determined in Mankiw's DAD-DAS model by a conventional *Phillips Curve* augmented to include the role of expected inflation, $E_{t-1}\pi_t$, and an exogenous inflation shock, v_t :

$$\pi_t = E_{t-1}\pi_t + \phi \cdot (Y_t - \bar{Y}_t) + v_t \quad (9)$$

where ϕ is a positive parameter.

When adaptive expectations (4) is substituted into the Phillips Curve (9), we get Mankiw's dynamic aggregate supply or DAS curve:

$$\pi_t = \pi_{t-1} + \phi \cdot (Y_t - \bar{Y}_t) + v_t. \quad (10)$$

It follows that the slope of the DAS curve is $d\pi_t/dY_t = \phi > 0$. Note also that $Y_t = \bar{Y}_t$ and $\pi_t = \pi_{t-1} + v_t$ satisfy equation (10). Figure 2 shows three positively sloped DAS curves (DAS_O , DAS_R , and DAS_D) for three different predetermined values of π_{t-1} (π^* , π_R , and $-\rho$, respectively) and zero inflation shock ($v_t = 0$). It follows that the heights of these three DAS curves at the full-employment output must also be π^* , π_R , and $-\rho$, respectively, as shown. Specifically, point O , at which $Y_t = \bar{Y}_t$ and $\pi_t = \pi^*$, must be on DAS_O , which is the DAS curve when $\pi_{t-1} = \pi^*$. Point D , at which $Y_t = \bar{Y}_t$ and $\pi_t = -\rho$, must be on DAS_D , which is the DAS curve when $\pi_{t-1} = -\rho$. In short, when there are no shocks, the height of the DAS curve at full employment is necessarily equal to the previous period's inflation rate.

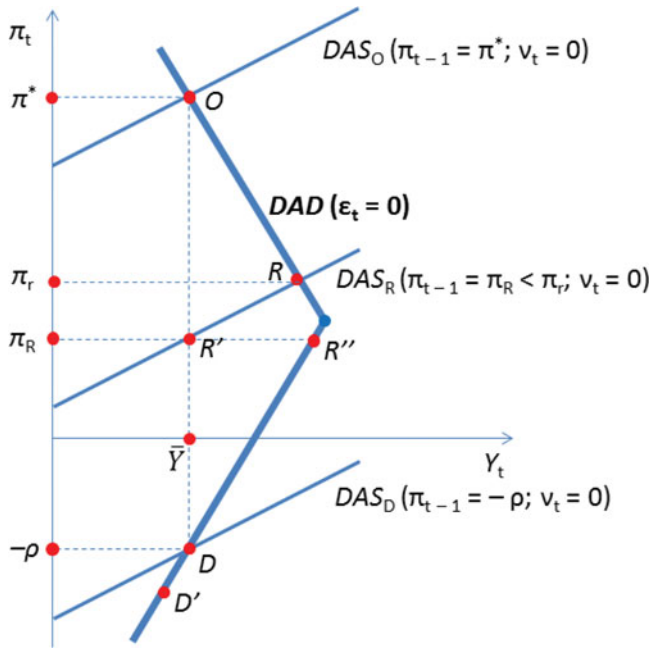


FIGURE 2 Points O and D are long-run equilibria, while point R is a short-run equilibrium. Both shocks are assumed zero. The DAS curves all satisfy equation (10) but for different levels of the previous period's inflation. Note that the height of a DAS curve at \bar{Y} , the full-employment output, is equal to the previous period's inflation rate.

Following Buttet and Roy (2014), we make the technical assumption that ϕ , the slope of the DAS curve, is smaller than $1/\alpha$, the slope of the positively sloped segment of the DAD curve (with ZLB binding): $1/\alpha > \phi$.

Short-Run and Long-Run Equilibrium

A *short-run equilibrium* at any period t is graphically represented by the intersection of the DAD and DAS curves for period t . Figure 2 shows three short-run equilibria (at O , R , and D) for the same DAD curve and three different DAS curves. Let us consider these three equilibria one by one.

Unless otherwise specified, we assume that (a) there are no demand or inflation shocks ($\epsilon_t = v_t = 0$ for all t), (b) the full-employment output is constant ($\bar{Y}_t = \bar{Y}$), and (c) the parameters of the model ($\alpha, \phi, \rho, \theta_\pi, \theta_Y, \pi^*$, and \bar{Y}) are constant. We do this to focus on the dynamic forces of change that are internal or endogenous to the economy (as opposed to change caused by shocks and parameter changes). As we saw in our subsection on the kinked DAD curve, under these assumptions, the DAD curve does not shift over time. Therefore, let the DAD curve in figure 2 be the economy's DAD curve in all periods.

Suppose the economy is in short-run equilibrium in period $t - 1$ at R'' in figure 2.¹⁷ Therefore, $\pi_{t-1} = \pi_R$. Recall from our subsection on the DAS curve that under our no-shocks and constant-parameters assumptions, the height of the DAS curve at the full-employment output, \bar{Y} , is equal

to the predetermined rate of the previous period's inflation. Therefore, the DAS curve in period t must be DAS_R and the short-run equilibrium in period t must therefore be at R with inflation at $\pi_t = \pi_r > \pi_R = \pi_{t-1}$. This example illustrates the internal dynamics of the DAD-DAS model whereby change can occur (in this case, from R'' at $t - 1$ to R at t) even though there are no shocks or parameter changes. (The reader can check that the short-run equilibrium at $t + 1$ will be somewhere between R and O on the DAD curve and that the economy converges to O over time.)

Next, suppose $\pi_{t-1} = \pi^*$, which is the central bank's inflation target. Therefore, the DAS curve in period t must be DAS_O . Therefore, the short-run equilibrium in period t must be at O because, as we saw in our subsections on the kinked DAD curve and the DAS curve, point O lies on both DAD and DAS_O . Consequently, $\pi_t = \pi^* = \pi_{t-1}$. In other words, O is a *long-run equilibrium*, which is a short-run equilibrium that repeats forever, as long as there are no shocks or parameter changes. Following Buttet and Roy (2014), we will refer to O as the *orthodox long-run equilibrium*. It can be checked that D is also a long-run equilibrium. Following Buttet and Roy (2014), we will refer to D as the *deflationary long-run equilibrium*.

Now that we have seen a short-run equilibrium, R , and two long-run equilibria, O and D , we can state the stability results established by Buttet and Roy (2014). They show that if $\pi_{t-1} > -\rho$, then in subsequent periods inflation and output will converge to π^* and \bar{Y} , respectively. That is, if inflation in some period exceeds the negative of the natural real interest rate, there is nothing to worry about as long as there are no shocks and no parameter changes: the economy will converge to the orthodox long-run equilibrium at O . On the other hand, if $\pi_{t-1} < -\rho$, then in subsequent periods, inflation and output will decrease indefinitely, moving southwest along the DAD curve away from D . This is the *deflationary spiral*, an especially undesirable outcome.¹⁸

Having discussed the model's equilibrium and stability properties, we next explain how a fall in the natural real interest rate, which is Summers's definition of secular stagnation, affects inflation and output in the short and long runs.

THE EFFECTS OF A DECREASE IN THE NATURAL RATE

As the natural real interest rate ρ does not appear in equation (10), it is clear that changes in ρ cannot shift the DAS curve. Similarly, ρ does not appear in equation (7), implying that changes in ρ cannot shift the negatively sloped segment of the DAD curve, which applies when the ZLB is not binding. We therefore have the following lemma:

Lemma 1: *When the ZLB is not binding in equilibrium (short-run or long-run), a decline in the natural real interest rate has no effect on the DAD and DAS curves. Therefore, the short-run equilibrium values of output and inflation are unaffected.*

This result, when coupled with the relevant monetary policy rule $i_t = \pi_t + \rho + \theta_\pi \cdot (\pi_t - \pi^*) + \theta_Y \cdot (Y_t - \bar{Y}_t)$, implies that for any decrease (respectively, increase) in the natural rate, ρ , the nominal interest rate decreases (respectively, increases) by the same percentage-point amount. Substituting adaptive expectations (4) into the Fisher equation (3) then yields the same result for the real interest rate. In an informal sense, it is this full adjustment of the two interest rates, i_t and r_t , to changes in ρ that makes it unnecessary for either output or inflation to adjust. There are limits, however, to the adjustment of the nominal interest rate to a falling natural rate. Although ρ can decrease indefinitely, the nominal interest rate cannot: it cannot fall below zero. Once i_t has been driven down to zero by repeated decreases in ρ , the economy will reach the ZLB.

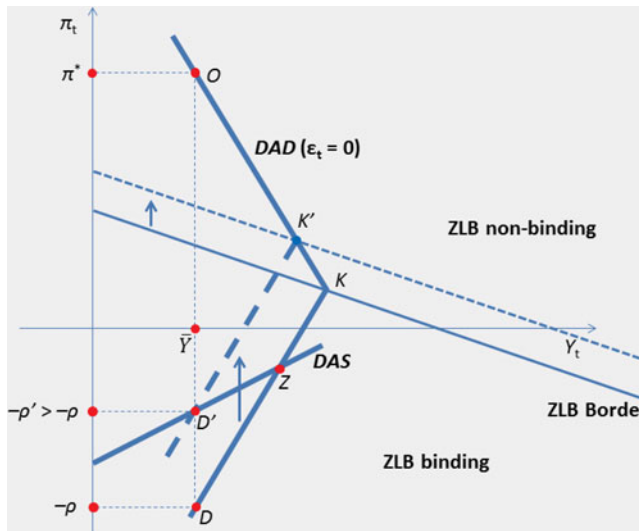


FIGURE 3 The effect of a fall in the natural rate ($\rho \downarrow$) on output and inflation. It is assumed that $\varepsilon_t = 0$ and $\bar{Y}_t = \bar{Y}$ for all t . A falling ρ has no effect on the orthodox long-run equilibrium, O , but moves the deflationary long-run equilibrium from D to D' , thereby bringing the deflationary equilibrium closer to the orthodox equilibrium. The short-run equilibrium moves from Z to D' , implying decreases in both output and inflation. Had the natural real interest rate fallen even slightly below ρ , a deflationary spiral would have been initiated.

For the ZLB case, note that the natural real interest rate ρ *does* appear in the equations for the ZLB border (6) and the positively sloped segment of the DAD curve (8). It is straightforward to check that a decrease in ρ shifts both the ZLB border (6) and the positively sloped segment of the DAD curve upward, as shown by the dashed lines in figure 3. It is also clear from equation (8) that any decrease (respectively, increase) in ρ leads to an *equal* upward (respectively, downward) shift in the rising segment of the kinked DAD curve. The effects of these shifts on short-run equilibrium are shown in figure 3. The economy is initially at Z but then moves to D' . We therefore have the following lemma:

Lemma 2: *When the ZLB on the nominal interest rate is binding in equilibrium (short-run or long-run), a decrease in the natural real interest rate ($\rho \downarrow$) leads to decreases in both output and inflation in the short run.*

To sum up, we have shown that in the short run, a fall in the natural real interest rate has no impact on inflation and output when the ZLB is not binding, but leads to declines in inflation and output when the ZLB is binding.

Next, we show that a decrease in the natural real interest rate brings the (unstable) deflationary long-run equilibrium closer to the (stable) orthodox long-run equilibrium, and thereby increases the likelihood that a demand and/or inflation shock might push the economy from the (stable) orthodox long-run equilibrium into a deflationary spiral.

Proposition 1: *A decrease in the natural real interest rate makes an economy more vulnerable to a deflationary spiral in the sense that the minimum size, in absolute value, that a demand or inflation shock must be in order to be big enough to initiate a deflationary spiral becomes smaller when the natural real interest rate decreases.*

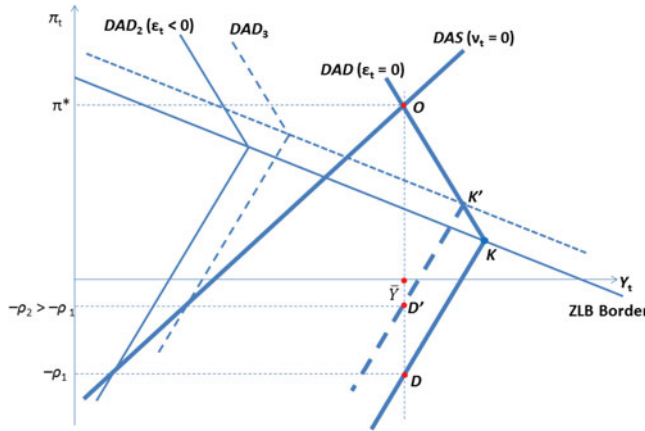


FIGURE 4 The natural real interest rate and the deflationary spiral-demand shock. A fall in the natural real interest rate makes the economy more vulnerable to a deflationary spiral. It is assumed that $v_t = 0$ and $\bar{Y}_t = \bar{Y}$ for all t . When the natural real interest rate decreases, the adverse demand shock that can tip the economy into a deflationary spiral becomes smaller. When $\rho = \rho_1$, a deflationary spiral can be initiated by the DAD curve moving to the left of DAD_2 . On the other hand, when $\rho = \rho_2 < \rho_1$, the DAD curve must move only to the left of DAD_3 .

As in figure 1, let the DAD curve in figure 4 initially be OKD . Note that the DAD and DAS curves intersect at O , indicating that the economy is initially at the orthodox long-run equilibrium with $Y_t = \bar{Y}$ and $\pi_t = \pi^*$. Now, consider a decrease in the natural or long-run real interest rate from ρ_1 to $\rho_2 < \rho_1$. As in figure 3, the DAD curve shifts from OKD to $OK'D'$. The deflationary long-run equilibrium shifts from D to D' , coming closer to O , the orthodox long-run equilibrium.

From equations (7) and (8), for the negatively sloped and positively sloped segments of the kinked DAD curve, and from equation (6) for the ZLB border, it is straightforward to check that an adverse demand shock (a decrease in ε_t) shifts the kinked DAD curve to the left and leaves the ZLB border unaffected.¹⁹ Therefore, when $\rho = \rho_1$, if there is a big enough negative demand shock, the DAD curve could shift just to the left of DAD_2 in figure 4, and thereby precipitate a deflationary spiral by reducing the inflation rate below $-\rho_1$.²⁰ On the other hand, when $\rho = \rho_2$, the demand shock must only be big enough to shift the DAD curve to the left of DAD_3 . In other words, a fall in the natural real interest rate makes the economy more vulnerable to a deflationary spiral because it reduces the size of the adverse demand shock that is just big enough to start a deflationary spiral.

We now consider the effect of an inflation shock in figure 5. Recall from the section on the DAS curve that a negative inflation shock ($v_t < 0$) leads to a vertically downward shift of the DAS curve. When $\rho = \rho_1$, a big enough negative (i.e., *favorable*) inflation shock could take the DAS curve to somewhere below DAS_2 , which would reduce inflation below $-\rho_1$, and thereby initiate a deflationary spiral. However, when $\rho = \rho_2 < \rho_1$, a *smaller* inflation shock would be able to initiate a deflationary spiral because the DAS curve must be pushed down only somewhere below DAS_3 .

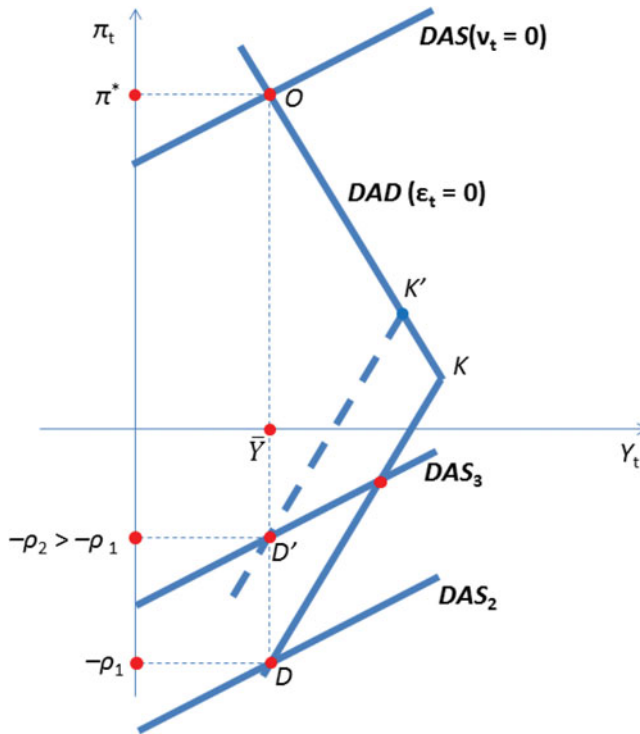


FIGURE 5 The natural real interest rate and the deflationary spiral-inflation shock. A fall in the natural real interest rate makes the economy more vulnerable to a deflationary spiral. It is assumed that $\varepsilon_t = 0$ and $\bar{Y}_t = \bar{Y}$ for all t . When the natural real interest rate decreases from ρ_1 to ρ_2 , the deflationary long-run equilibrium moves from D to D' . When $\rho = \rho_1$, a favorable inflation shock can initiate a deflationary spiral by moving the DAS curve to below DAS_2 . On the other hand, when $\rho = \rho_2 < \rho_1$, the DAS curve must drop only to below DAS_3 .

Finally, consider an extreme scenario where the natural real interest rate decreases to such an extent that $\pi^* = -\rho$. Then, the two long-run equilibria collapse into the same long-run equilibrium. In figure 6, this case is represented by the DAD curve AOE_2 . Here, O is still the long-run equilibrium in the sense that an economy at O remains at O in the absence of shocks, parameter changes, and policy changes. However, this long-run equilibrium is neither stable nor unstable. If $\pi_{t-1} > \pi^* = -\rho_2$, then the economy will converge to O (in the absence of any further shocks, parameter changes, and policy changes). However, if $\pi_{t-1} < \pi^* = -\rho_2$, then a deflationary spiral will take the economy away from O . If ρ falls further to ρ_3 , then $-\rho_3 > \pi^*$ and the DAD curve becomes AK_3E_3 in figure 6. In this case, there are *no long-run equilibria*. For all values of π_{t-1} , the economy will be in a deflationary spiral (in the absence of any further shocks, parameter changes, and policy changes).

As we will show in the next section, the good news is that this extreme case can be avoided by steadily raising the central bank's inflation target (π^*) whenever the natural real interest rate falls, thereby ensuring that $-\rho$ would never catch up to π^* .

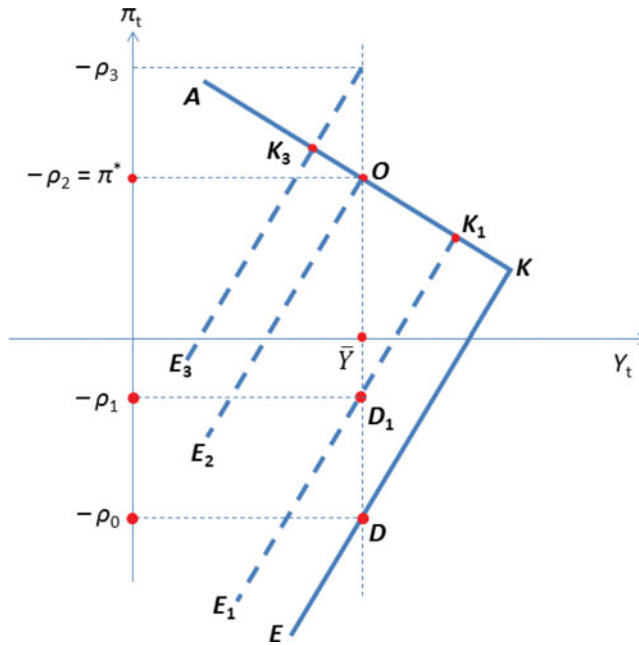


FIGURE 6 If ρ falls to $-\pi^*$ or below, the economy has either one neither-stable-nor-unstable long-run equilibrium or no long-run equilibrium. It is assumed that $\varepsilon_t = v_t = 0$ and $\bar{Y}_t = \bar{Y}$ for all t . As the natural real interest rate falls from ρ_0 to $\rho_1 < \rho_0$ to ρ_2 to ρ_3 , the DAD curve shifts from AKE to AK_1E_1 to AOE_2 to AK_3E_3 . For each of the first two, there are two long-run equilibria: O , which is stable, and the other unstable. For AOE_2 , O is the only long-run equilibrium, and it is neither stable nor unstable. When the DAD curve is AK_3E_3 , there is no long-run equilibrium; the deflationary spiral is the only possible outcome.

RAISING THE INFLATION TARGET TO NEUTRALIZE DECREASES IN THE NATURAL RATE

We have just seen in figures 4 and 5 that a decline in the natural real interest rate brings the (unstable) deflationary long-run equilibrium closer to the (stable) orthodox long-run equilibrium, thereby increasing the danger that a demand or inflation shock might push the economy from the orthodox long-run equilibrium into a deflationary spiral. An obvious solution is to move the orthodox long-run equilibrium in response to every move of the deflationary long-run equilibrium in such a way that a constant distance is maintained between the two.

While a decrease in ρ raises the positively sloped segment of the DAD curve (8) by the same percentage-point amount and leaves the negatively sloped segment (7) unaffected, it can be easily checked that an increase in π^* raises the negatively sloped segment by the same percentage-point amount and leaves the positively sloped segment unaffected. Therefore, as shown in figure 7, if ρ decreases by Δ percentage points, and simultaneously, π^* increases by Δ percentage points, then the DAD curve will shift from DAD (or, OKD) to DAD_2 (or, $O'K''D'$). Therefore, while the deflationary long-run equilibrium will move from D to D' , the orthodox long-run equilibrium will move from O to O' . In this way, an increase in the central bank's inflation target insulates

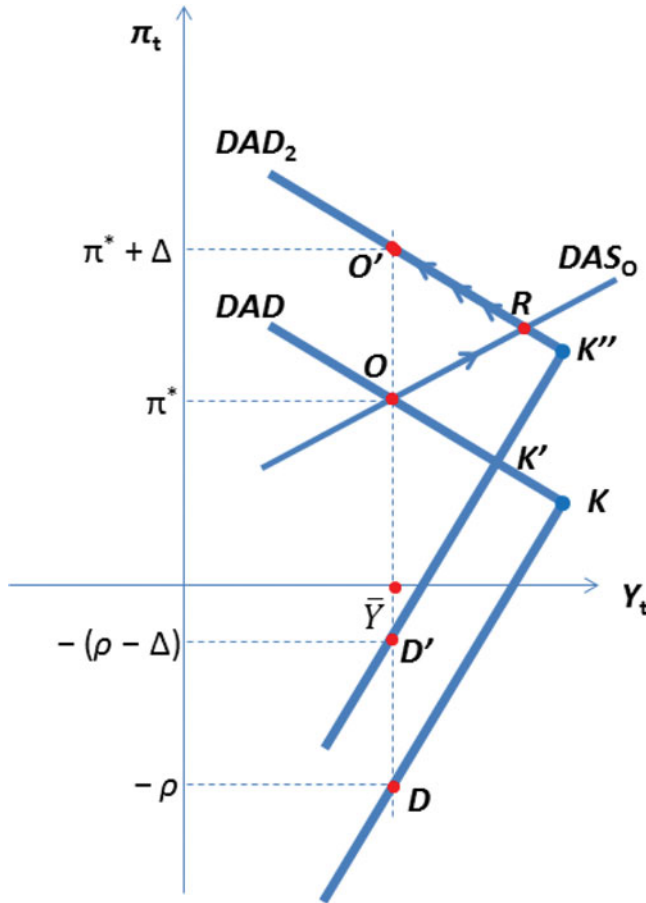


FIGURE 7 An effective way to neutralize a fall in the natural real interest rate is to raise the central bank's inflation target. It is assumed that $\varepsilon_t = v_t = 0$, $\bar{Y}_t = \bar{Y}$ for all t , and $\Delta > 0$. When the natural real interest rate decreases from ρ to $\rho - \Delta$, the deflationary long-run equilibrium moves from D to D' , coming closer to the stable long-run equilibrium, O , and increasing the chance that a shock would push the economy into a deflationary spiral. However, an increase in the central bank's target inflation from π^* to $\pi^* + \Delta$ maintains the distance between the two long-run equilibria and therefore does not allow the economy to become more vulnerable to a deflationary spiral when the natural real interest rate decreases.

the economy from the increased risk of a deflationary spiral associated with a decrease in the natural real interest rate.

We thus have the following proposition:

Proposition 2: *Although a decrease in the natural real interest rate increases an economy's vulnerability to a deflationary spiral, the threat would be neutralized if the central bank raises its target inflation rate by the same percentage-point amount as the decrease in the natural real interest rate.*

To see the irony in this result, consider an economy that is safely ensconced at the orthodox long-run equilibrium with $Y_t = \bar{Y}$ and $\pi_t = \pi^*$, and then imagine steady decreases in ρ . As we saw in lemma 1, output and inflation would be unaffected, and no danger would be apparent. Nevertheless, the probability that a sudden shock would push the economy into a deflationary spiral would be rising all the while, in an invisible, subterranean way. Therefore, our analysis suggests that decreases in the natural real interest rate should be counteracted with matching increases in the central bank's inflation target even when output is at full employment and inflation satisfies the central bank's inflation target.

Another notable point is that a negative natural real interest rate makes it possible for an economy to be in a deflationary spiral even when there is no deflation! The natural real interest rate could very well drop to *negative* levels, as indeed our estimates in the section on motivating evidence suggest it did in the post-2009 United States. Therefore, $-\rho$, the critical rate of inflation at which a deflationary spiral is triggered, could be positive. In such a situation, inflation could be positive and yet be low enough to trigger a deflationary spiral!

As the estimates in Laubach and Williams (2003) and our own estimates in this article suggest, the natural real interest rate has been falling in the United States since the 1980s, but nobody saw any reason for worry at the time because output and inflation were doing fine. Our lemma 1 explains why decreases in the natural real interest rate did not affect the economy back then: the ZLB on the nominal interest rate was not binding before December 2008. Our proposition 1 argues that the decrease in the natural real interest rate back in the pre-December 2008 period was making the economy more and more vulnerable to a deflationary spiral even though there was no visible impact on output and inflation at the time. As our proposition 2 argues, matching increases in the Fed's inflation target should have been implemented in the placid period before 2008.

Put another way, central banks should question the strategy of keeping inflation stable over the long run and consider instead a strategy of keeping the nominal interest rate stable over the long run. To see why, recall that $Y_t = \bar{Y}$ and $\pi_t = \pi^*$ in the stable orthodox long-run equilibrium. Then, from (2), the real interest rate in this equilibrium is $r_t = \rho$, and by (5), the nominal interest rate is $i_t = r_t + \pi_t = \rho + \pi^*$. In line with our argument that decreases in ρ should be matched by equal increases in π^* , it follows that $\rho + \pi^*$ should be kept constant. In other words, instead of keeping inflation stable over the long run, central banks should consider keeping the nominal interest rate stable over the long run.

These are new ideas in intermediate macroeconomics that follow from a standard model in a standard textbook. These ideas can be taught to intermediate macroeconomics students using standard graphical and algebraic techniques.

Finally, let us briefly consider how an economy that is in the orthodox long-run equilibrium would adjust over time if and when the central bank raises its inflation target.²¹ Suppose at time t the DAD curve is OKD in figure 7, the DAS is DAS_O , and the economy is therefore at the (stable) orthodox long-run equilibrium at O . Therefore, at t , output is $Y_t = \bar{Y}$, and inflation is $\pi_t = \pi^*$. At time $t + 1$, the DAS curve would still be DAS_O . (Recall from our subsection on the DAS curve that the height of the DAS curve at the full-employment output is the previous period's inflation, assuming no shocks.) However, the DAD curve at $t + 1$ would be DAD_2 , following the simultaneous decrease in ρ by Δ percentage points and an equal increase in π^* . Therefore, the short-run equilibrium at $t + 1$ would be at R , with $Y_{t+1} > \bar{Y} = Y_t$ and $\pi_{t+1} > \pi^* = \pi_t$.

In subsequent periods, it can be shown, by following the reasoning by which we showed the movement of the economy from R'' to R in figure 2 in our subsection on the DAS curve,

that the economy will over time move from R toward O' , the new (stable) orthodox long-run equilibrium, along $O'K''$. In short, when the central bank raises its inflation target, the economy will go immediately from O to R and then gradually from R to O' . Output will rise above (and then return to) the full-employment level. Inflation will rise gradually from π^* to $\pi^* + \Delta$. As for the interest rates, it can be shown that the real interest rate will fall from $r_t = \rho$ in the initial orthodox long-run equilibrium, by more than Δ percentage points, to $r_{t+1} < \rho - \Delta$ and then steadily increase to the new stable level of $\rho - \Delta$. The nominal interest rate will also fall from $i_t = \rho + \pi^*$ in the initial orthodox long-run equilibrium and then steadily increase to return to the unchanged level of $(\rho - \Delta) + (\pi^* + \Delta) = \rho + \pi^*$.

CONCLUSION

We have argued that changes in the natural real interest rate have important implications for the conduct of macroeconomic stabilization. We have made our argument using the DAD-DAS model in Mankiw (2013, ch. 15), modified to incorporate the ZLB on nominal interest rates. One key prescription for monetary policy that emerges from our analysis is that a prudent central bank should raise its target inflation rate when the natural real interest rate decreases, irrespective of whether the ZLB is binding or not. A higher target inflation insulates the economy from adverse demand shocks and favorable inflation shocks that could trigger a deflationary spiral.

We end by discussing two issues that have been mentioned in recent debates but are not discussed in our article. First, Summers (2013) has argued that attempts to fight negative real interest rates by raising the target inflation will lead to asset price bubbles and related financial instability. Krugman (2013) and Kocherlakota (2014) have argued that a separate set of policies (called macroprudential policies), aimed directly at the regulation of financial markets, are appropriate ways of addressing Summers's concerns.

Finally, both Summers (2013) and Krugman (2013) have argued in favor of prolonged fiscal stimulus as a response to secular stagnation. While aware that such fiscal stimulus would require government borrowing and increasing levels of government debt, both have argued that when real interest rates are lower than the growth rate of real GDP, prolonged government borrowing may be possible without any increase in the debt-to-GDP ratio, and should therefore be considered safe.

While we accept the importance of the issues summarized in the last two paragraphs, we were unable to address them formally in the model that we have used in this article. Our goal throughout has been to take a topic that is at the center of current macroeconomic debate (i.e., how to conduct monetary policy when real rates are decreasing) and show that a formal analysis of it can be made accessible to undergraduates without requiring them to learn a whole new model.

NOTES

1. See Summers (2014a) and the collection of papers in Teulings and Baldwin (2014).
2. Current editions of prominent intermediate macroeconomics textbooks (e.g., Mankiw 2013; Blanchard and Johnson 2013; Jones 2011; and Mishkin 2011) all discuss the ZLB, and all make the point that expansionary fiscal policy works at the ZLB, whereas expansionary monetary policy (at least of the conventional kind) does not. However, these textbooks do not explain the implications for

macroeconomic stabilization of changes in the natural real interest rate. Carlin and Soskice (2015, Section 3.3.3) provide a detailed account of the deflationary spiral.

3. The positively sloped segment of the DAD curve captures the idea that falling inflation is a special nightmare at the ZLB. As nominal interest rates cannot be reduced any further, any decline in inflation means an increase in the real interest rate, which in turn reduces aggregate demand and output.
4. Several central banks, including the Swiss National Bank, the European Central Bank, and the Danish National Bank, have recently adopted a negative nominal interest rate policy and as a result, the zero lower bound is not a firm lower bound. Negative nominal interest rates are not an issue for our analysis, however, because the only assumption needed for our theoretical results to carry through is that nominal interest rates are bounded from below. The floor value, whether positive or negative, is inconsequential. The negative nominal interest rates charged by banks reflect the cost of storage, and because of competitive pressure, this cost is unlikely to become much greater than 1 percent of deposit, thereby creating a floor on nominal interest rates (Cecchetti and Schoenholtz 2014).
5. In the extreme scenario where the natural real interest rate has fallen into negative territory, the economy could experience a deflationary spiral even though current inflation is positive (and presumably low).
6. Fiscal stimulus also can reverse a deflationary spiral, as shown in Buttet and Roy (2014), but tax cuts and/or increases in government purchases necessarily require increases in government borrowing, which may not always be an available option, especially in a weak economy and especially if the government has already piled up so large a debt that private lenders would be leery of lending it more. Therefore, there is a need to avoid getting into a deflationary spiral in the first place, and to avoid a deflationary spiral tomorrow, it is necessary to ensure that today's inflation stays above the negative of the natural real interest rate, which is the danger level.
7. Chadha and Perlman (2014), who analyzed the Gibson Paradox, note that in the presence of an uncertain natural rate, the need to stabilize the banking sector's reserve ratio can lead to persistent deviations of the market rate of interest from its natural level and consequently long-run swings in the price level.
8. Note that real interest rates have been declining in the United States since the 1980s with steady decreases in the estimated natural real interest rate, but central banks have not raised their inflation targets during that period. (Even the Bank of Japan's recent move in this direction was to increase its inflation target from 2 percent to a mere 3 percent.) This unwillingness must be reexamined in the light of our article.
9. In his address to the National Association for Business Economics and elsewhere, Summers (2014b, 67, 69) cautioned that even though the ZLB is not technically binding, low nominal and real interest rates undermine financial stability in various ways. The financial stability channel is not present in our article.
10. Eichengreen (2014) emphasized four different causes of secular stagnation in his review essay: slower growth of technological progress (Gordon 2014); stagnant aggregate demand (Summers 2014b; Krugman 2014); the failure of countries like the United States to invest in infrastructure, education, and training; and finally atrophy of skills caused by long-term unemployment and forgone on-the-job training (Crafts 1989; Gordon and Krenn 2010). It would be very hard (let alone undesirable) to write an article which encompasses all aspects of secular stagnation, so we focus on analyzing only one aspect of secular stagnation.
11. A recent International Monetary Fund (IMF) report (IMF 2014) cited three main reasons for the decline in real rates since the mid-1980s: (a) higher saving rates in emerging market economies, (b) greater demand for safe assets reflecting the rapid reserve accumulation of emerging market economies as well as increased riskiness of equity relative to bonds, and (c) a sharp and persistent decline in investment rates in advanced economies since the global financial crisis. All three factors lead to greater saving propensities and lower investment propensities.
12. Knut Wicksell (1898/1936, 102) offered the following definition for the natural real interest rate: "There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods." More recently, Kocherlakota (2014) referred to the natural real interest rate as the mandate-consistent real interest rate.

13. Laubach and Williams (2003) used maximum likelihood to estimate changes in the natural real interest rate over time, while the equilibrium results in Barsky, Justiniano, and Melosi (2014) are derived from solving a dynamic utility-maximization problem. We believe that the technical and critical thinking skills needed to fully understand these two modeling techniques are beyond the skill set that students possess when they take their intermediate macroeconomics courses at most colleges.
14. There are potential econometric issues with estimating changes in the natural rate using equation (1), such as co-integration of the variables, which could affect our estimates for the natural rate of interest. For the sake of space, however, and because our article proposes an innovation for intermediate macroeconomics courses, we do not discuss econometric issues related to estimating changes in the natural real interest rate here. Rather, we leave this discussion for an upper elective course on econometrics or time series analysis.
15. Note that an increase in the real interest rate leads to a decrease in aggregate demand, as in the standard IS curve. For the graphical analysis in the rest of the article, we will make the simplifying assumption: $\bar{y}_t = \bar{y}$ for all t .
16. The negative feedback loop between output and inflation is the mechanism that leads to a deflation-induced depression, as previously explained by Fisher (1933) and Krugman (1998). In normal times, when nominal interest rates are positive, the central bank can afford to cut interest rates following a negative demand shock to provide short-run stimulus to the economy. When the ZLB is binding, however, cutting rates is not feasible, and real interest rates spike up as a result of lower inflation. Higher real interest rates in turn depress the economy further, which put further pressure on real rates, which depress the economy further, and so on and so forth.
17. The DAS curve through R'' has not been drawn for simplicity.
18. To see the logic behind the unstable nature of the deflationary long-run equilibrium, D , and the deflationary spiral, see pages 46 and 47 of Buttet and Roy (2014). We saw above in figure 2 how an economy starting at R'' moves to R in the next period and further towards O in subsequent periods. It is straightforward to see the workings of the deflationary spiral by repeating that analysis, but starting at D' instead of R'' . Buttet and Roy (2014) emphasize the need to keep inflation above $-\rho$, and discuss how fiscal and monetary policy can be used (a) to stop inflation from falling below $-\rho$ and thereby precipitating a deflationary spiral, and (b) to raise inflation above $-\rho$ after it has already fallen below that level, thereby ending the deflationary spiral. They show that as the only way out of a deflationary spiral once it has begun is fiscal stimulus. Now, fiscal stimulus usually involves a tax cut or an increase in government purchases or both, and this usually requires an increase in government borrowing. And such borrowing, especially in conditions of economic weakness, may not be possible, especially for a government that has already borrowed a lot and is, therefore, treated warily by private lenders. That is why it is crucial that $\pi_{t-1} < -\rho$ be avoided at all costs.
19. See figure 2 of Buttet and Roy (2014) for a more detailed explanation.
20. Recall our discussion of the stability result in Buttet and Roy (2014): if inflation falls below the negative of the natural real interest rate, the economy will thereafter be in a deflationary spiral (if there are no further shocks or parameter changes), with output and inflation falling repeatedly.
21. A detailed discussion of this dynamic adjustment is provided in Mankiw (2013, 449–53).

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