HOMEWORK - DUE 11/15/2017

MATH 511

(1) Show (and explain why) the definition of β^α agrees with the usual definition for natural numbers when α and β are finite.

(2) Given any set A, the *power set* $\mathcal{P}(A)$ is the set of all subsets of A. In other words,

$$\mathcal{P}(A) = \{S \mid S \subseteq A\}.$$

  (a) If $A = \{a, b, c\}$, list all elements in $\mathcal{P}(A)$.
  (b) Show the general fact that $|\mathcal{P}(A)| = 2^{|A|}$ by constructing a natural bijection $\{0,1\}^A \overset{\approx}{\to} \mathcal{P}(A)$.

(3) Use cardinal arithmetic to show:
  (a) $8^{\aleph_0} = \mathfrak{c}$
  (b) If $A \subseteq B$ with $|A| = \aleph_0$ and $|B| = \mathfrak{c}$, then $|B \backslash A| = \mathfrak{c}$.
  (c) $\mathfrak{c}^{\aleph_0} = \mathfrak{c}$
  (d) $(\aleph_0)^{\aleph_0} = \mathfrak{c}$ (Hint: You may want to use inequalities to show this.)

(4) Compute the size of the following sets:
  (a) $\mathbb{C}^3$ (the set of all complex numbers)
  (b) The set of all $m \times n$ matrices with rational coefficients
  (c) The set of all functions from $\mathbb{Z}^2$ to $\{1, 2, 3, 4\}$
  (d) The set of all functions from $\mathbb{Z}^2$ to $\mathbb{Z}^3$

(5) Let E be the set of all theoretically possible English language books. (A book does not have to actually be written to be in E. Any finite collection of letters/words is an element of E.) Show that E is countable.

Date: November 1, 2017.