

## HOMEWORK - DUE 11/8/2017

MATH 511

Note: We did (versions of) several of these problem in class last Wednesday 10/25.

- (1) Show that there are the same “number” of integers as even integers.
- (2) Watch the video on Hilbert’s Hotel on the homework webpage. In Hilbert’s infinite hotel, there are countably many rooms and no vacancies. Show that one can accommodate a countably infinite number of new guests, despite not having any vacant rooms. Can you find multiple ways of accommodating the new guests?
- (3) Let  $a, b \in \mathbb{R}$  with  $a < b$ . Construct a bijection  $[0, 1] \xrightarrow{\approx} [a, b]$ . Construct a bijection  $(0, 1) \xrightarrow{\approx} (a, b)$ . Construct a bijection  $[0, 1) \xrightarrow{\approx} (0, 1]$ .
- (4) Prove that  $\mathbb{R} \approx (0, 1)$ . (*Hint:* Start with any function  $(a, b) \rightarrow \mathbb{R}$  that is a bijection.)
- (5) Show that  $[0, 1] \approx (0, 1]$ . (*Hint:* Let  $A \subset [0, 1]$  be a countable subset, and map every element in  $A$  to the next element.)
- (6) Use problems 3-5 to show that all intervals in  $\mathbb{R}$  are equivalent as sets.
- (7) Rewrite your (corrected) solutions to Test 2. You may talk with others, and you are encouraged to talk with me, but you should independently write up all work at the end. Turn in the new solutions along with the old test, though the revised solutions should not reference your original work.