1.39. A set A of real numbers is said to be *bounded from above* if there exists a number M such that $x \leq M$ for every x in A. (Such a number M is called an *upper bound* of M.)

(*a*) Suppose A and B are sets which are bounded from above with respective upper bounds M_1 and M_2. What can be said about the union and intersection of A and B?

(*b*) Suppose C and D are sets of real numbers which are unbounded. What can be said about the union and intersection of C and D?

(*a*) Both the union and intersection are bounded from above. In fact, the larger of M_1 and M_2 is always an upper bound for $A \cup B$, and the smaller of M_1 and M_2 is always an upper bound for $A \cap B$.

(*b*) The union of C and D must be unbounded, but the intersection could be either bounded or unbounded.

1.40. Restate the Principle of Mathematical Induction I and II in terms of sets, rather than assertions.

(*a*) *Principle of Mathematical Induction I*: Let S be a subset of $\mathbf{P} = \{1, 2, \ldots\}$ with two properties:

$$(1)\quad 1 \in S. \qquad (2)\quad \text{If } n \in S, \text{ then } n+1 \in S.$$

Then $S = \mathbf{P}$.

(*b*) *Principle of Mathematical Induction II*: Let S be a subset of $\mathbf{P} = \{1, 2, \ldots\}$ with two properties:

$$(1)\quad 1 \in S. \qquad (2)\quad \text{If } \{1, 2, \ldots, n-1\} \subseteq S, \text{ then } n \in S.$$

Then $S = \mathbf{P}$.

Homework 9/13
Text §1.3 p33: 6
§1.4 p41: 1, 4, 5
§1.5 p53: 1, 5a, 6a
and Circled problems below
1.42, 1.44, 1.51ae

Supplementary Problems

SETS AND SUBSETS

1.41. Which of the following sets are equal?

$A = \{x : x^2 - 4x + 3 = 0\}$ $\quad C = \{x : x \in \mathbf{P}, x < 3\}$ $\quad E = \{1, 2\}$ $\quad G = \{3, 1\}$

$B = \{x : x^2 - 3x + 2 = 0\}$ $\quad D = \{x : x \in \mathbf{P}, x \text{ is odd}, x < 5\}$ $\quad F = \{1, 2, 1\}$ $\quad H = \{1, 1, 3\}$

1.42. List the elements of the following sets if the universal set is $\mathbf{U} = \{a, b, c, \ldots, y, z\}$. Furthermore, identify which of the sets, if any, are equal.

$A = \{x : x \text{ is a vowel}\}$ $\quad C = \{x : x \text{ precedes f in the alphabet}\}$

$B = \{x : x \text{ is a letter in the word "little"}\}$ $\quad D = \{x : x \text{ is a letter in the word "title"}\}$

1.43. Let

$$A = \{1, 2, \ldots, 8, 9\}, \quad B = \{2, 4, 6, 8\}, \quad C = \{1, 3, 5, 7, 9\}, \quad D = \{3, 4, 5\}, \quad E = \{3, 5\}$$

Which of the above sets can equal a set X under each of the following conditions?

(*a*) X and B are disjoint. (*c*) $X \subseteq A$ but $X \not\subseteq C$.

(*b*) $X \subseteq D$ but $X \not\subseteq B$. (*d*) $X \subseteq C$ but $X \not\subseteq A$.

1.44. Consider the following sets:

$$\varnothing, \quad A = \{a\}, \quad B = \{c, d\}, \quad C = \{a, b, c, d\}, \quad D = \{a, b\}, \quad E = \{a, b, c, d, e\}.$$

Insert the correct symbol, $\subseteq$ or $\not\subseteq$, between each pair of sets:

(*a*) $\varnothing, A$ (*c*) A, B (*e*) B, C (*g*) C, D
(*b*) D, E (*d*) D, A (*f*) D, C (*h*) B, D

SET OPERATIONS

1.45. Let $\mathbf{U} = \{1, 2, 3, \ldots, 8, 9\}$ be the universal set and let:

$$A = \{1, 2, 5, 6\}, \quad B = \{2, 5, 7\}, \quad C = \{1, 3, 5, 7, 9\}$$

Find: (*a*) $A \cap B$ and $A \cap C$, (*b*) $A \cup B$ and $A \cup C$, (*c*) A^c and C^c.

1.46. For the sets in Problem 1.45, find: (*a*) $A \backslash B$ and $A \backslash C$, (*b*) $A \oplus B$ and $A \oplus C$.

1.47. For the sets in Problem 1.45, find: (*a*) $(A \cup C) \backslash B$, (*b*) $(A \cup B)^c$, (*c*) $(B \oplus C) \backslash A$.

1.48. Let $A = \{a, b, c, d, e\}$, $B = \{a, b, d, f, g\}$, $C = \{b, c, e, g, h\}$, $D = \{d, e, f, g, h\}$. Find:

(*a*) $A \cup B$ (*c*) $B \cap C$ (*e*) $C \backslash D$ (*g*) $A \oplus B$
(*b*) $C \cap D$ (*d*) $A \cap D$ (*f*) $D \backslash A$ (*h*) $A \oplus C$

1.49. For the sets in Problem 1.48, find:

(*a*) $A \cap (B \cup D)$ (*c*) $(A \cup D) \backslash C$ (*e*) $(C \backslash A) \backslash D$ (*g*) $(A \cap D) \backslash (B \cup C)$
(*b*) $B \backslash (C \cup D)$ (*d*) $B \cap C \cap D$ (*f*) $(A \oplus D) \backslash B$ (*h*) $(A \backslash C) \cap (B \cap D)$

1.50. Let A and B be any sets. Prove $A \cup B$ is the disjoint union of $A \backslash B$, $A \cap B$, and $B \backslash A$.

1.51. Prove the following:

(*a*) $A \subseteq B$ if and only if $A \cap B^c = \varnothing$ (*c*) $A \subseteq B$ if and only if $B^c \subseteq A^c$
(*b*) $A \subseteq B$ if and only if $A^c \cup B = U$ (*d*) $A \subseteq B$ if and only if $A \backslash B = \varnothing$
(Compare with Theorem 1.3.)

1.52. Prove the absorption laws: (*a*) $A \cup (A \cap B) = A$, (*b*) $A \cap (A \cup B) = A$.

1.53. The formula $A \backslash B = A \cap B^c$ defines the difference operation in terms of the operations of intersection and complement. Find a formula that defines the union $A \cup B$ in terms of the operations of intersection and complement.

1.54. (*a*) Prove: $A \cap (B \backslash C) = (A \cap B) \backslash (A \cap C)$.
(*b*) Give an example to show that $A \cup (B \backslash C) \neq (A \cup B) \backslash (A \cup C)$.

1.55. Prove the following properties of the symmetric difference:

(*a*) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ (Associative law)
(*b*) $A \oplus B = B \oplus A$ (Commutative law)
(*c*) If $A \oplus B = A \oplus C$, then $B = C$ (Cancellation law)
(*d*) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$ (Distributive law)