Math 512 Homework 8: Due April 17, 2013

April 10, 2013

1. Suppose an experiment has two possible outcomes, labeled by 0 and 1, where $P(0) = 1-p$ and $P(1) = p$. If the experiment is repeated, the new results are independent of the previous results (e.g. $P(01) = P(0)P(1)$). Calculate $I(0)$, $I(1)$, $I(000)$. Calculate the entropy in a single symbol, and use this to calculate the entropy contained in n symbols.

2. Suppose $A = \{0, 1, 2\}$, with each symbol having probability of $1/3$. Find the entropy contained in n symbols, assuming each symbol is independent of the previous symbols.

3. Suppose $A = \{0, 1, 2, \ldots, m-1\}$, and all symbols occur with equal probability. Find the entropy H in 1 symbol (in units bits/symbol). Describe an algorithm for asking yes/no questions that will always determine a number in A within $\lceil H \rceil$ questions. Here $\lceil H \rceil$ is H rounded up to the next integer (e.g. $\lceil 1.05 \rceil = 2$).

4. Explain, in your own words, how someone's personal calendar with the following data actually contains very little "information."

	Sun	Mon	Tues	Wed	Thur	Fri	Sat
Week 1	No Work	Work	Work	Work	Work	Work	No Work
Week 2	No Work	Work	Work	Work	Work	Work	No Work
Week 3	No Work	Work	Work	Work	Work	Work	No Work
Week 4	No Work	Work	Work	Work	Work	Work	No Work

5. (Added Thurs 4/11)
   (a) Let $A = \{1, 2, 3, 4\}$, with $p(1) = \frac{1}{2}$, $p(2) = \frac{1}{4}$, $p(3) = \frac{1}{8}$, $p(4) = \frac{1}{8}$. Calculate the entropy.
   (b) Let $A = \{1, 2, 3, \cdots\} = \mathbb{N}$, with $p(1) = \frac{1}{2}$, $p(2) = \frac{1}{4}$, $p(3) = \frac{1}{8}$, $\ldots$, $p(n) = 2^{-n}, \ldots$. Calculate the entropy. You may use the following series: $\displaystyle\sum_{i=0}^{\infty} \frac{i}{2^i} = 2$.