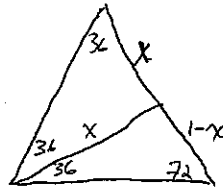
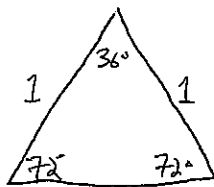


MATH 514: HOMEWORK DUE 4/9

- (1) Use the Rational Root Test to show that $\sqrt[3]{2}$ is irrational.
- (2) Let $n \in \mathbb{Z}$ (i.e. n is an integer). Use the Rational Root Test to show that: $\sqrt{n} \in \mathbb{Z}$ if and only if $\sqrt{n} \in \mathbb{Q}$.
- (3) Find a rational root of $3x^3 + x^2 + x - 2$. Use this root to show the polynomial is reducible over \mathbb{Q} .
- (4) Show that $x^6 + 1$ has no rational roots, but it is reducible over \mathbb{Q} .
Hint: Think of the polynomial $(x^2)^3 + 1 = y^3 + 1$, where $y = x^2$
- (5) Prove that $\cos(20^\circ)$ is not constructible. Conclude that the angle 60° is constructible, but it can not be trisected using a finite number of compass/straightedge constructions.
- (6) (a) Find an exact value for $\cos(72^\circ)$.
Hint: Use a 36° - 72° - 72° isosceles triangle, as shown in the picture.
 (b) Using part (a), prove (algebraically) that a regular pentagon can be constructed using compass and straightedge.
(You do not need to find the geometric construction.)



- (7) (Bonus) Show that if you are allowed an *infinite* number of compass/straightedge constructions, you can trisect any angle.