MATH 514: HOMEWORK 9
DUE 4/25

(1) Problems 8.4.1 - 8.4.3 on p. 186 from the textbook.

Here is a further explanation of what they are asking, in case you don't understand the textbook's questions.. Reflection in the unit circle, as discussed in class and as drawn in Figure 8.5, is given by the function $f(z) = 1/\bar{z}$. The goal of these 3 problems is to understand that this reflection f swaps the hyperbolic line given by $x = 1$ and the hyperbolic line given by the semi-circle with endpoints $z = 0$ and $z = 1$ (which is $(0, 0)$ and $(1, 0)$). Question 8.4.1 just asks for intuitive reasons; your answer will be an explanation of why this makes sense. For Question 8.4.2, assume $z + \bar{z} = 2$ and $w = 1/\bar{z}$; show that $w + \bar{w} = 2w\bar{w}$. For Question 8.4.3: assume $w + \bar{w} = 2w\bar{w}$ and $z = 1/\bar{w}$; show that $z + \bar{z} = 2$. Note that $\bar{\bar{z}} = z$.

(2) In the upper-half space model for hyperbolic geometry, consider the points $A = -2 + 2i$ and $B = 2 + 2i$, which we considered in class on 4/11.

(a) Recall the hyperbolic distance $d_H(A, B)$, either by redoing the integral or by looking it up in your notes.

(b) Determine the endpoints A_∞ and B_∞ on the x-axis of the hyperbolic line $\overleftrightarrow{AB}$.

(c) Calculate $|\ln[A, B; A_\infty, B_\infty]|$ via the formula

$$[z_1, z_2; z_3, z_4] = \left(\frac{z_1 - z_3}{z_2 - z_3}\right) \Big/ \left(\frac{z_1 - z_4}{z_2 - z_4}\right).$$

(*Note: Doing the division by hand may take some effort but is worthwhile. You can also use WolframAlpha.com if you scroll down for "Alternate Forms".*)

(d) Find a transformation $f(z) = \dfrac{az + b}{cz + d}$ that maps the line $\overleftrightarrow{AB}$ to the line $x = 0$.

(e) Using your f from the previous part, calculate $f(A_\infty), f(A), f(B), f(B_\infty)$.

(f) Calculate $|\ln[f(A), f(B); f(A_\infty), f(B_\infty)]|$.

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