2. "The sum of two numbers is twenty-five" and "twice the first number added to the second number totals thirty-two." Let x be the first number and y be the second number. $x = 7$, $y = 18$

3. "Tom has \$6 more than Alice" and "together, they have \$40." Let x be the amount of money that Tom has and y be the amount Alice has. $x = 23$, $y = 17$

4. "Bill and Jessica together have \$25" and "Jessica has \$12." Let x be the amount of money that Bill has and y be the amount that Jessica has. $x = 13$, $y = 12$

5. "An envelope of \$1 and \$5 bills contains thirty bills" and "the money in the envelope is worth \$70." Let x be the number of \$1 bills and y be the number of \$5 bills. $x = 20$, $y = 10$

6. "An envelope of \$10 and \$20 bills contains eight bills" and "the money in the envelope is worth \$110." Let x be the number of \$10 bills and y be the number of \$20 bills. $x = 5$, $y = 3$

7. "A small theater sold tickets for all one hundred seats" and "the box office receipts of \$650 came from adult tickets at \$10 and child tickets at \$5." Let x be the number of adult tickets sold and y be the number of child tickets sold. $x = 30$, $y = 70$

8. "A movie theater sold tickets for three hundred seats" and "the box office receipts of \$2400 came from adult tickets at \$9 and child tickets at \$6." Let x be the number of adult tickets sold and y be the number of child tickets sold. $x = 200$, $y = 100$

9. "A corn and beet farmer planted 225 acres of crops" and "he planted twice as many acres of corn as acres of beets." Let x be the number of acres of corn and y be the number of acres of beets. $x = 150$, $y = 75$

10. "A stock and bond speculator invested \$10,000 in the market" and "she invested three times as much in stocks as in bonds." Let x be the amount in stocks and y be the amount in bonds. $x = 7500$, $y = 2500$

Solve each system by graphing. If the solution is not unique, identify the system as "inconsistent" or "dependent."

You may use a graphing calculator if permitted by your instructor.

11. $\begin{cases} x + y = 6 \\ x - y = 2 \end{cases}$

12. $\begin{cases} -x + y = 2 \\ x + y = 4 \end{cases}$

13. $\begin{cases} 2x + y = 8 \\ x = 3 \end{cases}$

14. $\begin{cases} x + 2y = 10 \\ y = 4 \end{cases}$

15. $\begin{cases} x - y = 4 \\ -x + 2y = -6 \end{cases}$

16. $\begin{cases} 2x - y = 2 \\ x + 2y = 6 \end{cases}$

17. $\begin{cases} x + y = 10 \\ -x - y = 10 \end{cases}$

18. $\begin{cases} -2x + 4y = -16 \\ x - 2y = 4 \end{cases}$

19. $\begin{cases} x + y = 10 \\ -x - y = -10 \end{cases}$

20. $\begin{cases} 2x - 4y = 16 \\ -x + 2y = -8 \end{cases}$

Solve each system by the substitution method. If the solution is not unique, identify the system as "inconsistent" or "dependent."

21. $\begin{cases} x + 2y = 10 \\ y = 3 \end{cases}$

22. $\begin{cases} 2x + y = 8 \\ x = 2 \end{cases}$

23. $\begin{cases} 2x + y = 20 \\ x + y = 15 \end{cases}$

24. $\begin{cases} x + y = 12 \\ x + 2y = 14 \end{cases}$

25. $\begin{cases} 5x + 2y = 30 \\ 2x + y = 10 \end{cases}$

26. $\begin{cases} 2x + 3y = 25 \\ x - y = 5 \end{cases}$

27. $\begin{cases} 3x + 2y = 30 \\ x - y = -5 \end{cases}$

28. $\begin{cases} 2x + y = 20 \\ x + 3y = 15 \end{cases}$

29. $\begin{cases} -2x + 2y = -20 \\ x - y = 10 \end{cases}$

30. $\begin{cases} 3x - 2y = 30 \\ -6x + 4y = 30 \end{cases}$

Solve each system by the elimination method. If the solution is not unique, identify the system as "inconsistent" or "dependent."

31. $\begin{cases} x + y = 11 \\ 2x + 3y = 30 \end{cases}$

32. $\begin{cases} 3x + 2y = 30 \\ x + y = 13 \end{cases}$

33. $\begin{cases} 3x + y = 15 \\ x + 2y = 10 \end{cases}$

34. $\begin{cases} x + 3y = 30 \\ 2x + y = 10 \end{cases}$

35. $\begin{cases} x + 2y = 14 \\ 3x + 4y = 36 \end{cases}$

36. $\begin{cases} 2x + 3y = 30 \\ x - y = 10 \end{cases}$

6. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $a_{1,1}, a_{2,2}, a_{3,3}, a_{2,1}$

7. $(4 \quad 5 \quad 6 \quad 7)$; $a_{1,3}$ **8.** $(2 \quad 3 \quad 4 \quad 5 \quad 6)$; $a_{1,4}$

9. $\begin{pmatrix} 9 \\ 8 \\ 7 \\ 6 \\ 5 \end{pmatrix}$; $a_{2,1}, a_{4,1}$ **10.** $\begin{pmatrix} 2 \\ 8 \\ 3 \\ 7 \end{pmatrix}$; $a_{2,1}, a_{3,1}$

Find the augmented matrix representing the system of equations.

11. $\begin{cases} x + 2y = 2 \\ 3x + 4y = 12 \end{cases}$ **12.** $\begin{cases} 2x - y = 10 \\ -3x + 4y = 60 \end{cases}$

13. $\begin{cases} -4x + 3y = 84 \\ 5x - 2y = 70 \end{cases}$ **14.** $\begin{cases} -x + 2y = 2 \\ 2x - 3y = 6 \end{cases}$

15. $\begin{cases} 3x - 2y = 24 \\ x = 6 \end{cases}$ **16.** $\begin{cases} 4x - 3y = 24 \\ y = 8 \end{cases}$

17. $\begin{cases} 5x - 15y = 30 \\ -4x + 12y = 24 \end{cases}$ **18.** $\begin{cases} 12x - 4y = 36 \\ -15x + 5y = 45 \end{cases}$

19. $\begin{cases} x = 20 \\ y = 30 \end{cases}$ **20.** $\begin{cases} y = 18 \\ x = 12 \end{cases}$

Find the system of equations represented by the augmented matrix.

21. $\left(\begin{array}{cc|c} 1 & 1 & 9 \\ 0 & 1 & 4 \end{array}\right)$ **22.** $\left(\begin{array}{cc|c} 1 & 0 & -3 \\ 1 & 1 & 5 \end{array}\right)$

23. $\left(\begin{array}{cc|c} -4 & 3 & -60 \\ 1 & -2 & 20 \end{array}\right)$ **24.** $\left(\begin{array}{cc|c} 3 & 4 & 24 \\ 1 & 2 & 6 \end{array}\right)$

25. $\left(\begin{array}{cc|c} 1 & -3 & -70 \\ 1 & 1 & 10 \end{array}\right)$ **26.** $\left(\begin{array}{cc|c} 1 & 1 & 5 \\ 2 & 3 & 7 \end{array}\right)$

27. $\left(\begin{array}{cc|c} 2 & 1 & 6 \\ 1 & 2 & -6 \end{array}\right)$ **28.** $\left(\begin{array}{cc|c} 3 & 1 & -24 \\ 1 & 3 & 24 \end{array}\right)$

29. $\left(\begin{array}{cc|c} 20 & -15 & 60 \\ -16 & 12 & -48 \end{array}\right)$ **30.** $\left(\begin{array}{cc|c} 20 & -15 & 60 \\ -16 & 12 & 48 \end{array}\right)$

Carry out the row operation on the matrix.

31. $R_1 \leftrightarrow R_2$ on $\left(\begin{array}{cc|c} 3 & 4 & 24 \\ 5 & 6 & 30 \end{array}\right)$

32. $R_1 \leftrightarrow R_2$ on $\left(\begin{array}{cc|c} 8 & 7 & 56 \\ 6 & 5 & 60 \end{array}\right)$

33. $R_1 - R_2 \to R_1$ on $\left(\begin{array}{cc|c} 8 & 7 & 56 \\ 6 & 5 & 60 \end{array}\right)$

34. $2R_1 \to R_1$ on $\left(\begin{array}{cc|c} 3 & 4 & 24 \\ 5 & 6 & 30 \end{array}\right)$

35. $R_1 - 3R_2 \to R_1$ on $\left(\begin{array}{cc|c} 5 & 6 & 30 \\ 1 & 2 & 18 \end{array}\right)$

36. $R_1 - 3R_2 \to R_1$ on $\left(\begin{array}{cc|c} 6 & 5 & 60 \\ 2 & 2 & -4 \end{array}\right)$

37. $R_1 - R_2 \to R_2$ on $\left(\begin{array}{cc|c} 6 & 6 & -12 \\ 6 & 5 & 60 \end{array}\right)$

38. $R_1 - R_2 \to R_1$ on $\left(\begin{array}{cc|c} 5 & 6 & 30 \\ 4 & 8 & 72 \end{array}\right)$

39. $\frac{1}{8}R_2 \to R_2$ on $\left(\begin{array}{cc|c} 1 & -2 & -42 \\ 0 & 8 & 120 \end{array}\right)$

40. $\frac{1}{6}R_1 \to R_1$ on $\left(\begin{array}{cc|c} 6 & 0 & 420 \\ 0 & 1 & -72 \end{array}\right)$

Interpret each augmented matrix as the solution of a system of equations. State the solution or identify the system as "inconsistent" or "dependent."

41. $\left(\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -3 \end{array}\right)$ **42.** $\left(\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 8 \end{array}\right)$

43. $\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$ **44.** $\left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 1 \end{array}\right)$

45. $\left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$ **46.** $\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$

47. $\left(\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & 0 \end{array}\right)$ **48.** $\left(\begin{array}{cc|c} 1 & -4 & 6 \\ 0 & 0 & 0 \end{array}\right)$

49. $\left(\begin{array}{cc|c} 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right)$ **50.** $\left(\begin{array}{cc|c} 0 & 1 & 9 \\ 0 & 0 & 0 \end{array}\right)$

Solve each system by row-reducing the corresponding augmented matrix. State the solution or identify the system as "inconsistent" or "dependent."

If you have a graphing calculator with a RREF command, use it to check your row reduction.

51. $\begin{cases} x + y = 5 \\ x = 3 \end{cases}$ **52.** $\begin{cases} x + y = 8 \\ y = 6 \end{cases}$

53. $\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$ **54.** $\begin{cases} x - 2y = 6 \\ x + y = 3 \end{cases}$

➤ Solutions to Practice Problems

1. $R_2 \leftrightarrow R_3$

2.
$$\left(\begin{array}{cc|c} 3 & -6 & 12 \\ -5 & 10 & -14 \end{array}\right)$$
$$\tfrac{1}{3}R_1 \rightarrow \left(\begin{array}{cc|c} 1 & -2 & 4 \\ -5 & 10 & -14 \end{array}\right)$$
$$R_2 + 5R_1 \rightarrow \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 0 & 6 \end{array}\right)$$
Inconsistent, so *no* solution.

3. x_1, x_3, and x_4 are determined while x_2 is free. The solution is $\begin{cases} x_1 = -2t \\ x_2 = t \\ x_3 = 3 \\ x_4 = 4 \end{cases}$.

Exercises

Find the augmented matrix representing the system of equations.

1. $\begin{cases} x_1 + x_2 + x_3 = 4 \\ x_1 + 2x_2 + x_3 = 3 \\ x_1 + 2x_2 + 2x_3 = 5 \end{cases}$

2. $\begin{cases} x_1 + 5x_2 + 4x_3 = 6 \\ x_1 + x_2 + x_3 = 4 \\ 2x_1 + 3x_2 + 3x_3 = 9 \end{cases}$

3. $\begin{cases} 2x_1 - x_2 + 2x_3 = 11 \\ -x_1 + x_2 - 3x_3 = -12 \\ 2x_1 - 2x_2 + 7x_3 = 27 \end{cases}$

4. $\begin{cases} 4x_1 + 3x_2 - x_3 = 2 \\ 3x_1 + 3x_2 + 2x_3 = 9 \\ 2x_1 + x_2 - 3x_3 = -6 \end{cases}$

5. $\begin{cases} 2x_1 + x_2 + 5x_3 + 4x_4 + 5x_5 = 2 \\ x_1 + x_2 + 3x_3 + 3x_4 + 3x_5 = -1 \end{cases}$

6. $\begin{cases} 5x_1 + 2x_2 - 4x_3 + x_4 + 5x_5 = 7 \\ 3x_1 + x_2 - 3x_3 + x_4 + 3x_5 = 5 \end{cases}$

Find the system of equations represented by the augmented matrix.

7. $\left(\begin{array}{ccc|c} 4 & 3 & 2 & 11 \\ 3 & 3 & 1 & 6 \\ 1 & -2 & 3 & 13 \end{array}\right)$

8. $\left(\begin{array}{ccc|c} 3 & -2 & 5 & 23 \\ -1 & 1 & -3 & -12 \\ 2 & -2 & 7 & 27 \end{array}\right)$

9. $\left(\begin{array}{ccc|c} 2 & 1 & 1 & 7 \\ 2 & 2 & 1 & 6 \\ 3 & 3 & 2 & 10 \end{array}\right)$

10. $\left(\begin{array}{ccc|c} 5 & 1 & 3 & 20 \\ 1 & 1 & 2 & 6 \\ 4 & 1 & 3 & 17 \end{array}\right)$

11. $\left(\begin{array}{cccc|c} 8 & 3 & -2 & 19 & 15 \\ 3 & 1 & -1 & 7 & 6 \end{array}\right)$

12. $\left(\begin{array}{cccc|c} 6 & -2 & -4 & -2 & 36 \\ 2 & -1 & -10 & 5 & 6 \end{array}\right)$

Interpret each row-reduced matrix as the solution of a system of equations.

13. $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -4 \end{array}\right)$ **14.** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$

15. $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ **16.** $\left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$

17. $\left(\begin{array}{ccc|c} 1 & 0 & -1 & -5 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right)$

18. $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

Use an appropriate row operation or sequence of row operations to find the equivalent row-reduced matrix.

If your instructor permits, you may carry out the calculations on a graphing calculator using the ROWOPS program.

19. $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right)$ **20.** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \end{array}\right)$

21. $\left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$ **22.** $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \end{array}\right)$

23. $\left(\begin{array}{ccc|c} 2 & 4 & 0 & 6 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$ **24.** $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 5 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$

Solve each system of equations by the Gauss–Jordan method. If the solution is not unique, identify the system as "dependent" or "inconsistent."

If you have a graphing calculator with a RREF command, use it to check your row reduction.

25. $\begin{cases} x_1 + x_2 + x_3 = 2 \\ x_1 + 2x_2 + 2x_3 = 3 \\ x_1 + 3x_2 + 2x_3 = 1 \end{cases}$

26. $\begin{cases} 2x_1 + 3x_2 + x_3 = 4 \\ 3x_1 + 3x_2 + 2x_3 = 12 \\ x_1 + 2x_2 + x_3 = 4 \end{cases}$

27. $\begin{cases} 2x_1 + 2x_2 - x_3 = -5 \\ -2x_1 - x_2 + x_3 = 3 \\ 3x_1 + 4x_2 - x_3 = -8 \end{cases}$

28. $\begin{cases} 3x_1 - 4x_2 + 2x_3 = -15 \\ -x_1 + 2x_2 - x_3 = 6 \\ 4x_1 - 3x_2 + 2x_3 = -16 \end{cases}$

29. $\begin{cases} 2x_1 + x_2 - 2x_3 + x_4 = 2 \\ x_1 + x_2 + 2x_3 + x_4 = 5 \\ x_1 + x_2 + x_3 + x_4 = 4 \\ 2x_1 + 2x_2 + 3x_3 + x_4 = 8 \end{cases}$

30. $\begin{cases} x_1 - x_2 + x_3 + 2x_4 = 7 \\ x_1 - 2x_2 + x_3 + 2x_4 = 8 \\ 2x_1 + 2x_2 + x_3 + 3x_4 = 7 \\ x_1 - x_2 + x_3 + x_4 = 6 \end{cases}$

31. $\begin{cases} 2x_1 + 3x_2 + x_3 = 4 \\ 3x_1 + 5x_2 + 2x_3 = 12 \\ x_1 + 2x_2 + x_3 = 3 \end{cases}$

32. $\begin{cases} x_1 + 3x_2 + 2x_3 = 6 \\ x_1 + 2x_2 + 2x_3 = 3 \\ 2x_1 + 5x_2 + 4x_3 = 8 \end{cases}$

33. $\begin{cases} 4x_1 + 3x_2 + 2x_3 = 24 \\ x_1 + x_2 + 3x_3 = 7 \\ 5x_1 + 4x_2 + 5x_3 = 31 \end{cases}$

34. $\begin{cases} 5x_1 - 7x_2 + 3x_3 = -9 \\ -x_1 + x_2 - x_3 = 1 \\ 4x_1 - 5x_2 + 3x_3 = -6 \end{cases}$

35. $\begin{cases} x_1 + x_2 + 2x_3 + x_4 = 2 \\ 2x_1 + 2x_2 + 3x_3 + 3x_4 = 9 \\ 2x_1 + x_2 + 2x_3 + 2x_4 = 7 \\ x_1 + x_2 + x_3 + x_4 = 4 \end{cases}$

36. $\begin{cases} 3x_1 + 7x_2 + 4x_3 + 4x_4 = -7 \\ 7x_1 + 7x_2 + 4x_3 + 7x_4 = 3 \\ 4x_1 + 3x_2 + 2x_3 + 3x_4 = 2 \\ 3x_1 + 2x_2 + x_3 + 3x_4 = 4 \end{cases}$

37.
$$\begin{cases} x_1 - x_2 + x_4 = 1 \\ 2x_1 - 2x_2 + 2x_4 = 2 \\ x_1 - x_2 - x_3 - x_4 = 1 \\ 2x_1 - 2x_2 - x_3 = 1 \end{cases}$$

38.
$$\begin{cases} x_1 - x_3 + x_4 = 2 \\ x_2 + 2x_3 + x_4 = 0 \\ 2x_1 - 2x_2 - 6x_3 = 5 \\ x_1 + x_2 + x_3 + 2x_4 = 2 \end{cases}$$

39.
$$\begin{cases} x_1 + x_2 + x_3 + 2x_4 = 3 \\ x_1 - x_3 + x_4 = 2 \\ x_1 + 2x_2 + 3x_3 + 3x_4 = 4 \\ x_2 + 2x_3 + x_4 = 1 \end{cases}$$

40.
$$\begin{cases} x_1 + 2x_2 + x_3 - 2x_4 = -3 \\ 2x_1 + 4x_2 + 2x_3 - x_4 = 0 \\ x_1 + 2x_2 + x_3 + x_4 = 3 \\ x_1 + 2x_2 + x_3 = 1 \end{cases}$$

APPLIED EXERCISES

Formulate each situation as a system of linear equations. Be sure to state clearly the meaning of each variable. Solve using the Gauss–Jordan method. State your final answer in terms of the original question.

If you have a graphing calculator with a RREF command, use it to check your row reduction.

41. GENERAL: Plant Fertilizer A backyard garden needs 35 pounds of potash, 68 pounds of nitrogen, and 25 pounds of phosphoric acid. Three brands of fertilizer, GrowRite, MiracleMix, and GreatGreen, are available and contain the amounts of potash, nitrogen, and phosphoric acid per bag listed in the table. How many bags of each brand should be used to provide the required potash, nitrogen, and phosphoric acid?

(Pounds per Bag)	GrowRite	MiracleMix	GreatGreen
Potash	4	6	7
Nitrogen	5	10	16
Phosphoric acid	3	4	5

42. ATHLETICS: Nutrition A student athlete has decided to "bulk up" before the wrestling season by supplementing his weekly diet with an additional 325 grams of protein, 185 grams of fiber, and 110 grams of fat. If a hamburger contains 20 grams of protein, 10 grams of fiber, and 5 grams of fat; a cheeseburger contains 25 grams of protein, 10 grams of fiber, and 5 grams of fat; and a "sloppy-joe" contains 20 grams of protein, 15 grams of fiber, and 10 grams of fat, how many of each should he eat this week to meet his goal?

43. BUSINESS: Oil Well Ownership An accountant manages a limited-partnership that exploits oil-depletion tax advantages for wealthy clients, and sells memberships in the partnership for $5000, $10,000 or $25,000. If the partnership has seven hundred members and an investment value of $6 million, how many memberships of each amount are there? What is the greatest possible number of $25,000 members?

44. PERSONAL FINANCE: Financial Planning An international investment banker wishes to invest $150,000 in U.S. and German stocks and bonds. Stocks are not as secure as bonds, so he plans to spread his investments so that he has three times as much in U.S. bonds as in U.S. stocks and twice as much in German bonds as in German stocks. Furthermore, he intends to invest just $45,000 in stocks altogether. How much does he invest in each?

45. PERSONAL FINANCE: Income Taxes **a.** Find the federal, state, and city income taxes on a taxable income of $180,000 if the federal tax is 50% of the taxable income after first deducting the state and city taxes, the state tax is 17% of the taxable income after first deducting the federal and city taxes, and the city tax is 3% of the taxable income after first deducting the federal and state taxes. **b.** Although it appears that the taxpayer is facing a 70% nominal tax rate, use the actual taxes to determine the effective combined tax rate for this situation.