HOMEWORK DUE MONDAY 12/11

MATH 615

Homework: #3-#5. Numbers #1-#2 optional, depending on if you need more work with change of basis problems.

(1) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given in standard coordinates by the matrix

$$\begin{bmatrix} -1 & 6 \\ \frac{3}{2} & -1 \end{bmatrix}$$

Let $B = \{(1,0),(0,1)\}$ and $B' = \{(-2,1),(2,1)\}$.

  (a) Find the change of basis matrices $P_{BB'}$ and $P_{B'B}$ and use these to compute the matrix of T relative to B' (i.e. the above matrix is T_{BB}, and use $P_{BB'}$ and $P_{B'B}$ to find $T_{B'B'}$).
  (b) Use $T_{B'B'}$ to find the kernel and image of T.

(2) Let $T : \mathbb{P}_2 \to \mathbb{R}^2$ be given by

$$T(p) = \begin{bmatrix} p(0) \\ p(2) \end{bmatrix}$$

(e.g. if $p = a + bx$, then $p(4) = a + b(4) = a + 4b$.)

  (a) Find the matrix of T relative to the standard bases $B = \{1, x, x^2\}$ of $\mathbb{P}_2$, and $C = \{\mathbf{e}_1, \mathbf{e}_2\}$ of $\mathbb{R}^2$.
  (b) Find the matrix of T relative to the basis $A = \{1, 1+x, 1+x+x^2\}$ of $\mathbb{P}_2$ and $D = \{(1,1),(1,-1)\}$ of $\mathbb{R}^2$.

(3) Suppose that $T : \mathbb{P}_2 \to \mathbb{P}_2$ satisfies

$$T(1+x) = 3(x+x^2), \quad T(x+x^2) = -(x^2+1), \quad T(x^2+1) = 2(x+x^2) + (x+x^2).$$

Calculate $\operatorname{Ker} T$ and $\operatorname{Im} T$.
(Hint: Find the matrix of T relative to the basis $\{1+x, x+x^2, x^2+1\}$, calculate kernel and image relative to that basis, then rewrite the kernel and image as polynomials.)

(4) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfies

$$T(3,1) = 5(3,1), \quad T(0,2) = -1(0,2).$$

Find the matrix of T relative to the standard basis of $\mathbb{R}^2$.

(5) Suppose that a matrix $A, B \in \mathcal{M}_{n\times n}$ is diagonal; i.e. the entry in the i-th row, j-th column

$$A_{ij} = \begin{cases} a_i & i = j \\ 0 & i \neq j \end{cases} \qquad B_{ij} = \begin{cases} b_i & i = j \\ 0 & i \neq j \end{cases}.$$

Prove that AB is diagonal with $(AB)_{ii} = a_i b_i$.