MATH 616 HOMEWORK
DUE 3/26/18

(1) (p. 275: 2.12) Perform the Gram-Schmidt process on this basis for $\mathbb{R}^3$:
$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \right\}$$

(2) Let $V = \mathbb{R}^2$ with subspace W given by the line $x + 2y = 0$.
   (a) Find a basis for W.
   (b) Which vectors are in the subspace
   $$W^\perp = \{v \in V \mid \langle v, w \rangle = 0 \quad \forall w \in W\}?$$
   (c) For $\mathbf{v} = (0, 4)$, calculate the orthogonal projection $\mathrm{proj}_W\, \mathbf{v}$.
   (d) Verify that $\mathbf{v} - \mathrm{proj}_W\, \mathbf{v} \in W^\perp$.

(3) Let $V = \mathbb{R}^3$, and let W be the plane spanned by the vectors $\{1, 0, 0\}$ and $\{0, 1, 1\}$. What vector in W is closest to $\{2, 2, 2\}$? What vector in W is closest to $\{-5, 0, 2\}$?

(4) Let $\langle f, g \rangle = \int_0^{2\pi} f\, g\, dx$. Calculate $\langle \cos(kx), \sin(lx) \rangle$ and $\langle \cos(kx), \cos(lx) \rangle$, where k, l are positive integers.

(5) Let $f : [0, 2\pi] \to \mathbb{R}$ be the following piecewise function:
$$f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ 0 & \pi < x \le 2\pi \end{cases}$$
   (a) Calculate the Fourier coefficients a_0, a_n, b_n for $f(x)$.
   (b) Sketch a graph of $f(x)$ and the 5^{th} Fourier approximation
   $$S_5(x) = a_0 + \sum_{n=1}^{5} (a_n \cos(nx) + b_n \sin(nx)).$$

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