A simple treatment of the liquidity trap for Intermediate Macroeconomics courses

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Abstract

Leading undergraduate textbooks of intermediate macroeconomics such as Mankiw (2010) or Jones (2011) now include a simple reduced-form New Keynesian model of short-run dynamics, but without any analysis of how the zero lower bound on nominal interest rates affects the dynamics. The purpose of this paper is to show how instructors in intermediate macroeconomics courses can modify the aforementioned model to teach undergraduate students about the significance of the liquidity trap for economic performance and policy. This acquires additional significance because economies such as the United States and Japan have been close to the zero lower bound since 2008 and 1995, respectively. We show that the introduction of the non-negativity constraint on nominal interest rates dramatically affects the dynamics for output and inflation in both the short run and the long run. When the zero lower bound is binding, an additional long-run equilibrium exists that is unstable and can lead to a deflationary spiral. We find that both fiscal and monetary policy can keep an economy out of a deflationary spiral whereas only fiscal policy can end a deflationary spiral that has already begun.

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1 Introduction

Just a few years before the financial crisis hit, many economists and central bankers attributed a large part of the decline in macroeconomic volatility in the past thirty years – the Great Moderation period – to improved monetary policy (Bernanke, 2004) and an improved understanding of the economy’s inner workings (Lucas, 2003). The Great Recession of 2009 certainly challenged the neoclassical synthesis and led to a renewed research interest in academic circles and in Washington, D.C., on how to conduct policy in a liquidity trap, on the size of fiscal multipliers when nominal interest rates are zero (Blanchard et al., 2010; Christiano et al., 2009), and on new strategies for communicating monetary policy decisions (Eggertsson and Woodford, 2003; Bernanke et al., 2004; Woodford, 2010).

Much to our surprise, however, the treatment of the liquidity trap, including the effectiveness of monetary and fiscal policy, is still very scant in leading undergraduate textbooks for macroeconomics at the intermediate or advanced levels such as Mankiw (2010) and Jones (2011). The purpose of this paper is to present a simple reduced-form New Keynesian model that instructors can use in their intermediate or advanced level courses in macroeconomics to educate undergraduate students about the significance of the liquidity trap for economic performance and policy.

Reduced-form New Keynesian models of short-run economic fluctuations usually do not incorporate any non-negativity restrictions on nominal interest rates. They are built around three equations: an IS curve (the output gap depends on the real interest rate), a Phillips curve (inflation depends on the output gap and the last period’s expectation of this period’s inflation rate), and a Taylor rule (without a zero lower bound constraint). The cyclical and long-run properties of New Keynesian models, without the zero lower bound on nominal interest rates, are well known. In the short run, output and the real interest rate are pro-cyclical and, in the absence of shocks, all variables converge toward a unique stable equilibrium where the economy operates at full capacity and inflation is equal to the central bank’s inflation target.

The introduction of the zero lower bound changes the model’s dynamics in the short and long runs. The economy now has two long-run equilibria: (i) a stable equilibrium where nominal interest rates are positive and inflation is equal to the central bank’s target
and (ii) an unstable equilibrium such that the nominal interest rate is zero and even the slightest shock can set off a deflationary spiral\(^1\).

The multiplicity of equilibria leads to interesting questions about short-run dynamics. For a given set of parameter values and initial conditions, to which of the two equilibria would inflation and output converge? How large should a negative demand shock be to trigger a depression and are there equilibrium paths where deflation does not lead to depression? How effective are monetary and fiscal policy at preventing or stopping the deflationary spiral?

We find that answers to the first two questions depend on the sign of a simple expression, the sum of the inflation rate and the natural rate of interest. We show that, as long as the sum of the inflation rate and the natural real interest rate stays positive, the economy converges back to the long-run stable equilibrium, even when the zero lower bound is initially binding. On the other hand, a deflationary spiral starts when the sum of the inflation rate and the natural rate of interest becomes negative. One scenario that makes the sum of the inflation rate and the natural rate negative is a large contractionary demand shock that induces a sudden fall in the inflation rate. However, since this scenario has a small probability of occurrence, few equilibrium paths lead to a depression.

We also study the effectiveness of monetary and fiscal policy in dealing with the deflationary spiral. We distinguish between prevention and remedy, as proposed by Ben Bernanke in his 2002 speech about deflation (Bernanke, 2002), and ask two simple questions: (i) What can be done to keep an economy away from the deflationary spiral? (ii) Can policy get an economy out of a deflationary spiral, if it is already in one? We show that expansionary fiscal policy is an adequate answer to both questions, while expansionary monetary policy—specifically, an increase in the target inflation rate—is a partial

\(^1\)The negative feedback loop between output and inflation is the mechanism that leads to a deflation-induced depression, as previously explained by Fisher (1933) and Krugman (1998). In normal times, when nominal interest rates are positive, the central bank can afford to cut interest rates following a negative demand shock to provide short-run stimulus to the economy. When the zero-lower bound is binding, however, cutting rates is not feasible and real interest rates spike up as a result of lower inflation. Higher real rates in turn depress the economy further, which put further pressure on real rates, which depress the economy further, and so on and so forth.
Finally, we present model simulations to answer two quantitative questions. First, how large should a demand shock be to trigger a depression? Second, how aggressive should the policy response be to prevent or cure a depression? Given our choice of parameter values, it takes an output loss of at least 24 percent over four consecutive periods to trigger a deflation-induced depression. The likelihood of such large shocks is infinitesimal. The answer to the second question is also encouraging. Everything else equal, a modest rise in the central bank’s inflation target from 2 to 3 percent is enough to prevent the start of a deflationary spiral and only large fiscal stimulus packages (expressed as a fraction of the natural level of output) have a chance to get the economy out of the deflationary spiral, when it is already in one.

The remainder of the paper is organized as follows. In Sections 2 and 3, we introduce our model and characterize its long-run equilibria. We discuss the stability of these equilibria in Section 4. We study the policy responses to a deflationary environment in Section 5 and present some numerical simulation results in Section 6. Finally, we offer concluding remarks in Section 7.

2 The Model

Our goal here is to take a typical model of short-run macroeconomic dynamics from a standard undergraduate intermediate macroeconomics textbook and demonstrate how the analysis can be enriched if a non-negativity constraint on the nominal interest rate is added. The model that we have chosen to use is variously referred to as the dynamic AD-AS (or DAD-DAS) model in Mankiw (2010), the AS/AD model in Jones (2011), and the 3-equation (IS-PC-MR) model in Carlin and Soskice (2006), where PC refers to the Phillips curve and MR refers to the central bank’s monetary policy rule. This dynamic model has begun to supplement or even replace the static IS-LM model as the mainstay of short-run analysis in undergraduate macroeconomics textbooks. It is our belief that adding the zero lower bound to the teaching of short-run macroeconomic dynamics in undergraduate courses will increase the realism and relevance of the analysis because the interest rates used by central banks as their instruments of monetary policy have lately
been close to zero for a considerable length of time in several countries. For example, the Federal Funds Rate, which is the policy rate for the Federal Reserve in the United States, has been near zero since October 2008. The official discount rate in Japan has been close to zero since 1995.

In the DAD-DAS model in Mankiw (2010), equilibrium in the market for goods and services is given by

\[ Y_t = \bar{Y}_t - \alpha \cdot (r_t - \rho) + \epsilon_t \]  

(1)

where \( \bar{Y}_t \) denotes the natural or full-employment level of output, \( r_t \) is the real interest rate, \( \rho \) is the natural real interest rate, \( \alpha \) is a positive parameter representing the responsiveness of aggregate expenditure to the real interest rate, and \( \epsilon_t \) represents demand shocks. This equation is essentially the well-known IS curve of the IS-LM model, and it has no intertemporal dynamics. The shock \( \epsilon_t \) represents exogenous shifts in demand that arise from changes in consumer and/or business sentiment—the so-called “animal spirits”—as well as changes in fiscal policy. When the government implements a fiscal stimulus (an increase in government expenditure or a decrease in taxes), \( \epsilon_t \) is positive, whereas fiscal austerity makes \( \epsilon_t \) negative.

The ex-ante real interest rate in period \( t \) is determined by the Fisher equation and is equal to the nominal interest rate \( i_t \) minus the inflation expected for the next period:

\[ r_t = i_t - E_{t+1} \pi_t + 1 \]  

(2)

Inflation in the current period, \( \pi_t \), is determined by a conventional Phillips curve augmented to include the role of expected inflation, \( E_{t-1} \pi_t \), and exogenous supply shocks, \( \nu_t \):

\[ \pi_t = E_{t-1} \pi_t + \phi \cdot (Y_t - \bar{Y}_t) + \nu_t \]  

(3)

where \( \phi \) is a positive parameter.

Inflation depends on expected inflation because some firms set prices in advance. When these firms anticipate high inflation, they expect their own costs and their competitors’ prices to rise quickly. This expectation induces firms to raise their prices. Conversely, when firms expect low inflation, they forecast that their costs and competitors’ prices will rise only modestly. In this case, they keep price increases in check, which leads to low inflation.
The business cycle also has an effect. Inflation goes up when output rises above its natural level, as the parameter $\phi$ is positive. When the economy is booming and output rises above its natural level, firms experience increasing marginal costs and so they raise their prices. Conversely, when the economy is in recession and output is below its natural level, marginal costs fall and firms cut prices, which results in low inflation.

The inflation shock $\nu_t$ captures all other influences on inflation other than inflation expectations (which are captured in the term $E_{t-1}\pi_t$) and short-run economic conditions (which are captured in the term $\phi \cdot (Y_t - \bar{Y}_t)$). For example, an increase in the price of imported oil would mean a positive value for $\nu_t$.

Inflation expectations play a key role in both the Fisher equation (2) and the Phillips curve (3). As in Mankiw (2010), we assume that inflation in the current period is the best forecast for inflation in the next period. That is, agents have adaptive expectations:

$$E_{t}\pi_{t+1} = \pi_t.$$  \hspace{1cm} (4)

Note that adaptive expectations (4) turns the Fisher equation (2) into $r_t = i_t - \pi_t$ and the Phillips curve (3) into $\pi_t = \pi_{t-1} + \phi \cdot (Y_t - \bar{Y}_t) + \nu_t$. Thus, the Phillips curve becomes the sole source of intertemporal dynamics in this model.

Finally, we complete the description of the model with a rule for monetary policy. Dynamic New Keynesian models assume that the central bank sets a target for the nominal interest rate, $i_t$, based on the inflation gap and the output gap according to the Taylor rule (Taylor, 1993). Specifically, the DAD-DAS model in Mankiw (2010) assumes that the monetary policy rule is $i_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^*) + \theta_Y (Y_t - \bar{Y}_t)$, where the central bank’s inflation target ($\pi_t^*$) and its policy parameters $\theta_\pi$ and $\theta_Y$ are all non-negative. We, however, wish to explicitly incorporate the fact that nominal interest rates need to be non-negative. Therefore, our generalized monetary policy rule is:

$$i_t = \max\{0, \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^*) + \theta_Y (Y_t - \bar{Y}_t)\}.$$  \hspace{1cm} (5)

The main intermediate macroeconomics textbooks that discuss the dynamic model of short-run macroeconomics, all make this assumption of adaptive expectations. Recall that our goal is to show how the textbook treatment can accommodate the zero lower bound on nominal interest rates. The role of expectations has been analyzed in great detail in Eggertsson and Woodford (2003, 2004).
Note that although the central bank targets the nominal interest rate, $i_t$, its influence on the economy works through the real interest rate, $r_t$, as is clear from the IS curve (1). When inflation in period $t$ is equal to the central bank’s target ($\pi_t = \pi_t^*$) and output is at its natural level ($Y_t = \bar{Y}_t$), the last two terms in equation (5) are equal to zero, implying that the real interest rate is equal to the natural real interest rate ($r_t = \rho$). When inflation rises above the central bank’s target ($\pi_t > \pi_t^*$) or output rises above its natural level ($Y_t > \bar{Y}_t$), the Taylor rule engineers an increase in the real interest rate to bring down inflation and/or output. Conversely, if inflation falls below the central bank’s target ($\pi_t < \pi_t^*$) or output falls below its natural level ($Y_t < \bar{Y}_t$), the real interest falls to provide a stimulus to the economy. In this way, the monetary policy rule implements the so-called Taylor principle.

### 3 Long-Run Equilibria

For given values of the model’s period-$t$ parameters ($\alpha$, $\rho$, $\phi$, $\theta_{\pi t}$, $\theta_{Y t}$, $\pi_t^*$, and $\bar{Y}_t$), its period-$t$ shocks ($\epsilon_t$ and $\nu_t$), and the pre-determined inflation rate ($\pi_{t-1}$) for period $t - 1$, one can use the model’s five equations to solve for its five period-$t$ endogenous variables ($Y_t$, $r_t$, $i_t$, $E_t \pi_{t+1}$, and $\pi_t$). Once $\pi_t$ is determined, the process can be repeated for period $t + 1$, and so on and on.

To study the model’s dynamic properties algebraically—as opposed to numerically—we will simplify the model by assuming a constant target inflation rate: that is, $\pi_t^* = \pi^*$ for all $t$.

**Definition 1.** An equilibrium is a sequence for output, inflation, nominal and real interest rates $\{Y_t, \pi_t, i_t, r_t\}$ such that the model’s equations (1)–(5) are satisfied at all time $t$.

**Definition 2.** A long-run equilibrium is any equilibrium sequence such that, in the absence of shocks ($\epsilon_t = \nu_t = 0$ for all $t$), the inflation rate $\pi_t$ is constant.

It is then straightforward to identify the two long-run equilibria of the model. First, one can check that if $\pi_{t-1} = \pi^*$ and there are no shocks, then $\pi_{t+s} = \pi^*$ for all subsequent periods. We will refer to this long-run equilibrium as the orthodox equilibrium. As Mankiw (2010) has shown, in this long-run equilibrium, real output is equal to potential output.
in all periods, \( Y_{t+s} = \bar{Y}_{t+s} \); the real interest rate is equal to the natural real interest rate, \( r_{t+s} = \rho \); and the nominal interest rate is constant and equal to \( i_{t+s} = \rho + \pi^* \).\(^3\)

Second, one can check that, if \( \pi_{t-1} = -\rho \) and there are no shocks, then \( \pi_{t+s} = -\rho \) for all subsequent periods. We refer to this long-run equilibrium as the \textit{deflationary equilibrium}. In this long-run equilibrium, real output is equal to potential output, \( Y_{t+s} = \bar{Y}_{t+s} \); the real interest rate is equal to the natural rate of interest, \( r_{t+s} = \rho \); and the nominal interest rate is zero forever, \( i_{t+s} = 0 \).\(^4\)

4 Stability of Equilibrium

The issue of stability inevitably arises: How does the economy behave for arbitrary values of \( \pi_{t-1} \)? Will it converge to one of the two long-run equilibria? If so, which one?

To analyze the stability properties of our generalized DAD-DAS model, we need to solve for the model’s endogenous variables for the case in which the zero lower bound on the nominal interest rate is not binding and again for the case in which the zero lower bound is binding.

4.1 When the Zero Lower Bound is Non-Binding

In this case, the monetary policy rule (5) becomes

\[
i_t = \pi_t + \rho + \theta_{xt} \cdot (\pi_t - \pi_t^*) + \theta_{yt} \cdot (Y_t - \bar{Y}_t).
\]

Therefore, the economy is described by equations (1)–(4) and (6). Using these equations, it is straightforward, if tedious, to derive expressions for the model’s endogenous variables \( (Y_t, r_t, i_t, E_t\pi_{t+1} = \pi_t) \) in terms of the parameters of the model and the pre-determined

\(^3\)To verify that this is indeed a long-run equilibrium, one can check that, for any instant \( t, \pi_t = \pi^* \), \( Y_t = \bar{Y}_t \), \( r_t = \rho \), \( i_t = \rho + \pi^* \), and \( \pi_{t-1} = \pi^* \) satisfies equations (1)-(5) when the two shocks are at zero. To ensure that the orthodox long-run equilibrium does not run afool of the zero lower bound on the nominal interest rate we assume \( \rho + \pi^* > 0 \) or, equivalently \( \pi^* > -\rho \).

\(^4\)The DAD-DAS model in Mankiw (2010) does not have this deflationary long-run equilibrium. This is because it does not have the zero lower bound. Again, it is straightforward to verify that, for any instant \( t, \pi_t = -\rho \), \( Y_t = \bar{Y}_t \), \( r_t = \rho \), \( i_t = 0 \), and \( \pi_{t-1} = -\rho \) satisfies equations (1)-(5) when the two shocks are at zero.
inflation rate of the previous period ($\pi_{t-1}$). Specifically, it can be shown that equilibrium output at time $t$ is

$$Y_t = \bar{Y}_t + \frac{\alpha \theta_{\pi t} \cdot (\pi^*_t - \pi_{t-1} - \nu_t) + \epsilon_t}{1 + \alpha \cdot (\theta_{\pi t} \phi + \theta_{\pi Y t})},$$

(7)
equilibrium inflation is

$$\pi_t = \frac{(1 + \alpha \theta_{\pi Y t}) \cdot (\pi_{t-1} + \nu_t) + \alpha \theta_{\pi t} \phi \pi^*_t + \phi \epsilon_t}{1 + \alpha \cdot (\theta_{\pi t} \phi + \theta_{\pi Y t})},$$

(8)
and the equilibrium nominal interest rate is

$$i_t = \rho + \frac{(1 + \theta_{\pi t} + \alpha \theta_{\pi Y t})(\pi_{t-1} + \nu_t) - (1 - \alpha \phi)\theta_{\pi t} \pi^*_t + (\theta_{\pi Y t} + (1 + \theta_{\pi t})\phi) \epsilon_t}{1 + \alpha \cdot (\theta_{\pi t} \phi + \theta_{\pi Y t})}.$$

(9)

As the Fisher equation (2) and adaptive expectations (4) imply $r_t = i_t - \pi_t$, the equilibrium real interest rate can be derived from equations (8) and (9) above.\(^5\)

It follows from (7) that output ($Y_t$) is directly related to full-employment output ($\bar{Y}_t$), the demand shock ($\epsilon_t$), and the central bank’s target inflation ($\pi^*_t$), and inversely related to the previous period’s inflation ($\pi_{t-1}$), the inflation shock ($\nu_t$), the responsiveness of inflation to output in the Phillips Curve ($\phi$), and the responsiveness of the nominal interest rate to output in the monetary policy rule ($\theta_{\pi Y t}$).

It follows from (8) that inflation ($\pi_t$) is directly related to $\pi_{t-1}$, $\nu_t$, $\pi^*_t$, and $\epsilon_t$. We will see in section 5 that these comparative static results for inflation play an important policy role in the dynamic stabilization of the economy.

Figure 1 graphs the dynamic mapping from $\pi_{t-1}$ to $\pi_t$ for given values of all the parameters in equation (8) and shocks set at zero ($\epsilon_t = \nu_t = 0$). It can be checked that this graph—which we will call the inflation mapping curve—must intersect the 45-degree line at $\pi_{t-1} = \pi^*_t$, because $\pi_{t-1} = \pi^*_t$ implies $\pi_t = \pi_{t-1} = \pi^*_t$. This represents the orthodox long-run equilibrium of section 3.

Note also that the inflation mapping curve must be flatter than the 45-degree line as its slope is a positive fraction: that is, $\partial \pi_t / \partial \pi_{t-1} = (1 + \alpha \theta_{\pi Y t}) / (1 + \alpha \theta_{\pi Y t} + \alpha \theta_{\pi t} \phi) \in (0, 1)$. As is shown in Figure 1, this feature of the inflation mapping curve implies that the orthodox long-run equilibrium is stable.

\(^5\)Note the distinction between equilibrium at time $t$ and the long-run equilibrium defined in section 3.
Figure 1: **Dynamics without the zero lower bound on the nominal interest rate**

The dynamic mapping from $\pi_{t-1}$ to $\pi_t$ when there are no shocks ($\epsilon_t = \nu_t = 0$) and the zero lower bound on the nominal interest rate is non-binding is shown here. For any given value of $\pi_{t-1}$, the inflation mapping curve and the 45-degree line can be used to trace inflation in subsequent periods. The orthodox long-run equilibrium ($\pi_{t-1} = \pi^*$) can be seen to be stable.
For reasons discussed in section 4.2, we assume that $1 - \alpha \phi > 0$. It then follows from (9) that the nominal interest rate is directly related to $\rho$ and to all exogenous variables and shocks that are also directly related to inflation.

Note that these results prevail only when $i_t$, as given by (9), is non-negative. It can be checked that (9) implies

$$i_t \begin{cases} > 0 \text{ if and only if } & \pi_{t-1} \begin{cases} > \pi^c_{t-1}, \end{cases} \end{cases}$$

where

$$\pi^c_{t-1} \equiv \frac{(1 - \alpha \phi) \theta_{\pi t} \pi^*_t - (1 + \alpha \theta_{\pi t} \phi + \alpha \theta_{Y t}) \rho - (\theta_{\pi t} \phi + \theta_{Y t} + \phi) \epsilon_t}{1 + \theta_{\pi t} + \alpha \theta_{Y t}} - \nu_t$$

is the critical value of $\pi_{t-1}$, the previous period’s inflation rate, such that the equilibrium nominal interest rate is given by equation (9) when $\pi_{t-1} \geq \pi^c_{t-1}$, and is zero when $\pi_{t-1} \leq \pi^c_{t-1}$.

The line segment $BC$ in Figure 2 graphs the dynamic mapping from $\pi_{t-1}$ to $\pi_t$ in equation (8) for $\pi_{t-1} \geq \pi^c_{t-1}$ when there are no shocks and the central bank has a constant target inflation ($\pi^*_t = \pi^*$). Note, as in our discussion of Figure 1 above, that the coefficient of $\pi_{t-1}$ on the right hand side of equation (8) is a positive fraction, which explains why $BC$ is drawn flatter than the 45-degree line. At $\pi_{t-1} = \pi^*$, $BC$ intersects the 45-degree line, indicating the orthodox long-run equilibrium of section 3. It can be checked from (11) that, in the absence of shocks, $\pi^c_{t-1}$ is a convex combination of $\pi^*$ and $-\rho$. Our assumption that $\pi^* + \rho > 0$ then implies $\pi^* > \pi^c_{t-1} > -\rho$, as shown in Figure 2.

Convergence to the orthodox long-run equilibrium can also be shown algebraically. In the absence of shocks ($\epsilon_t = \nu_t = 0$), equation (8) reduces to

$$\pi_t - \pi_{t-1} = \frac{\alpha \theta_{\pi t} \phi}{1 + \alpha \theta_{\pi t} \phi + \alpha \theta_{Y t}} (\pi^*_t - \pi^{c*}_{t-1}).$$

Note that whenever the central bank’s target inflation rate, $\pi^*$, exceeds (respectively, is less than) the previous period’s inflation, $\pi_{t-1}$, the current period’s inflation rises (respectively, falls), while still remaining less (respectively, greater) than the target inflation. In other words, if there are no shocks and $\pi_{t-1} \geq \pi^c_{t-1}$, the inflation rate converges over time to the central bank’s target inflation rate.
Note that under adaptive expectations and in the absence of shocks, the Phillips curve (3) yields
\[ \phi \cdot (Y_t - \bar{Y}_t) = \pi_t - \pi_{t-1} = \frac{\alpha \theta_{\pi t} \phi}{1 + \alpha \theta_{\pi t} \phi + \alpha \theta_{Y t}} (\pi^* - \pi_{t-1}). \]
Therefore, the convergence of inflation to target inflation—established in the previous paragraph—implies the convergence of equilibrium output to the full-employment output.

Therefore, by the monetary policy rule, (5) or (6), the nominal interest rate set by the central bank converges to \( i^* = \pi^* + \rho > 0 \). And, by the Fisher equation (2) the real interest rate converges to \( \rho \).

To summarize, we have established the following:

**Proposition 1.** When \( \pi_{t-1} \geq \pi^c_{t-1} \), the equilibrium values of output, inflation, and the nominal interest rate at time \( t \) are given by equations (7), (8), and (9). Moreover, if there are no shocks, the economy converges to the orthodox long-run equilibrium.

### 4.2 When the Zero Lower Bound is Binding

We will now discuss the new dynamics that the zero lower bound on nominal interest rates adds to the textbook DAD-DAS model of Mankiw (2010).

It follows from equations (5) and (10) that, when \( \pi_{t-1} < \pi^c_{t-1} \), the zero lower bound on the nominal interest rate becomes binding: that is,
\[ i_t = 0. \tag{13} \]

Therefore, when \( \pi_{t-1} < \pi^c_{t-1} \), the economy is described by equations (1)–(4) and (13). Using these equations, it is straightforward, as before, to derive expressions for the endogenous variables \( (Y_t, r_t, i_t, E_t\pi_{t+1} = \pi_t) \) at time \( t \) in terms of the parameters of the model and the pre-determined inflation rate, \( \pi_{t-1} \), of the previous period. Specifically, equations (1)–(4) and (13) imply that equilibrium output at time \( t \) is
\[ Y_t = \bar{Y}_t + \frac{\alpha \cdot (\pi_{t-1} + \nu_t + \rho) + \epsilon_t}{1 - \alpha \phi} \tag{14} \]
and equilibrium inflation is
\[ \pi_t = \frac{1}{1 - \alpha \phi} (\pi_{t-1} + \nu_t) + \frac{\phi}{1 - \alpha \phi} (\alpha \rho + \epsilon_t). \tag{15} \]
To summarize, if $\pi_{t-1} < \pi_c^{t-1}$, the equilibrium values of output, inflation, and the nominal interest rate at time $t$ are given by equations (14), (15), and (13).

To guarantee the rather intuitive result that an increase in the demand shock ($\epsilon_t$) leads to an increase in output ($Y_t$), we assume $1 - \alpha \phi > 0$. Note that the coefficient of $\pi_{t-1}$ in (15) exceeds unity when $1 - \alpha \phi > 0$. This is why the segment $AB$ in Figure 2, which shows the dynamic link between $\pi_{t-1}$ and $\pi_t$ for $\pi_{t-1} \leq \pi_c^{t-1}$ when there are no shocks, is steeper than the 45-degree line.\(^6\)

The contrast between equation (7), when the zero lower bound is not binding, and equation (14), when the zero lower bound is binding, is striking. It follows from (14) that $\pi_{t-1}$, $\nu_t$, and $\phi$ are now directly related to output (in contrast to the inverse effects in equation (7)). Moreover, output is now directly related to the responsiveness of aggregate demand to the real interest rate ($\alpha$) and to the long-run real interest rate ($\rho$), and unrelated to all monetary policy tools ($\pi^*_t$, $\theta_{Yt}$, and $\theta_{\pi t}$). This ineffectiveness of monetary policy follows from the fact that the nominal interest rate is now at the zero lower bound and monetary policy is, therefore, unable to influence the nominal interest rate.

It follows from equation (15) that inflation is directly related to $\pi_{t-1}$, $\nu_t$, $\phi$, and $\epsilon_t$, as it is in equation (8), which describes inflation when the zero lower is not binding. However, $\alpha$ and $\rho$ now have a direct effect on inflation, and the monetary policy tools have no effect at all (for the reason discussed in the previous paragraph).

Subtracting $\pi_{t-1}$ from both sides of equation (15) yields

$$\pi_t - \pi_{t-1} = \frac{\alpha \phi}{1 - \alpha \phi} \cdot (\pi_{t-1} + \rho).$$

This confirms our earlier result that when there are no shocks and $\pi_{t-1} = -\rho$, the economy is in the deflationary long-run equilibrium of section 3.

However, equation (16) also reveals the unstable nature of the deflationary long-run equilibrium. If there are no shocks and if $\pi_{t-1} < -\rho$, equation (16) implies that inflation keeps falling forever: $\pi_{s+1} < \pi_s < -\rho$ for all $s \geq t$. And, by equation (14), the output gap $(Y_s - \bar{Y}_s)$ will be negative and decreasing, indefinitely. This is the dreaded deflationary spiral.

\(^6\)Also, it can be checked that equating the right hand sides of equations (8) and (15) yields $\pi_{t-1} = \pi_{t-1}^c$, thereby confirming the continuity of the mapping of $\pi_{t-1}$ to $\pi_t$ at $\pi_{t-1} = \pi_{t-1}^c$ in Figure 2.
By contrast, if there are no shocks and if \( \pi^c_{t-1} > \pi_{t-1} > -\rho \), inflation and the output gap both keep rising.

Therefore, we have established the following:

**Proposition 2.** Assume there are no shocks. The deflationary long-run equilibrium is unstable.

As inflation rises over time when \( \pi^c_{t-1} > \pi_{t-1} > -\rho \) and there are no shocks, equation (11) implies that inflation will, in finite time, exceed \( \pi^c_{t-1} \) and that, therefore, the zero lower bound on the nominal interest rate will switch from binding to non-binding.

Therefore, propositions 1 and 2 can be combined to yield the following:

**Proposition 3.** Assume there are no shocks from time \( t \) onwards. If \( \pi_{t-1} > -\rho \), the economy converges to the orthodox long-run equilibrium. If \( \pi_{t-1} = -\rho \), the economy stays in the deflationary long-run equilibrium. If \( \pi_{t-1} < -\rho \), the economy stays in a deflationary spiral, with both output and inflation decreasing without bound.

Proposition 3, which is graphically illustrated in Figure 2, helps explain why central banks rarely implement the Friedman rule as the optimal monetary policy. The Friedman rule, which calls for a small dose of anticipated deflation, takes the sum of inflation and natural rate of interest very close to zero, which makes the economy vulnerable to further shocks and raises the probability that the economy would plunge into a depression.

5 Policy Responses to Deflationary Spirals

Given that a deflationary spiral—with output decreasing without bound—is undesirable, (i) what can be done to keep an economy away from it, and (ii) what can be done to get an economy out of a deflationary spiral if it is already in one?\(^7\) We will show

\(^7\)None of the channels of unconventional monetary policy at the zero lower bound are at work in our model. Bernanke et al. (2004) discuss the policy tools that central banks can use when the zero lower bound is binding, including balance sheet expansion through “quantitative easing”; changing the composition of the Fed’s balance sheet through, for example, the targeted purchases of long-term bonds as a means of reducing the long-term interest rate; or using communication to shape public expectations about the future course of interest rates.
Figure 2: **Stability** $ABC$ represents the dynamic mapping from $\pi_{t-1}$ to $\pi_t$ when there are no shocks ($\epsilon_t = \nu_t = 0$). $AB$ represents an economy in which the zero lower bound on the nominal interest rate is binding, and $BC$ represents the same economy when the zero lower bound on the nominal interest rate is non-binding. For any given value of $\pi_{t-1}$, $ABC$ and the 45-degree line can be used to trace inflation in subsequent periods. The orthodox long-run equilibrium ($\pi_{t-1} = \pi^*$) can be seen to be stable, whereas the deflationary long-run equilibrium ($\pi_{t-1} = -\rho$) can be seen to be unstable.
that expansionary fiscal policy—that is, an increase in $\epsilon_t$ in the goods market’s equilibrium condition (1)—is an adequate answer to both questions, and expansionary monetary policy—that is, an increase in the central bank’s target inflation rate ($\pi^*$) in the monetary policy rule (5)—is an answer to (i).

5.1 Avoiding a Deflationary Spiral

From Proposition 3, we see that, if $\pi_{t-1} \geq -\rho$ and there are no shocks from period $t$ onwards, the economy will not enter a deflationary spiral. However, the possibility of adverse shocks in period $t$ cannot be wished away. We need to ask what can be done in period $t$ to neutralize adverse period-$t$ shocks that can cause a deflationary spiral from period $t+1$ onwards. We will consider two cases: Case 1A: $\pi_{t-1} \geq \pi^c_{t-1} > -\rho$, and Case 1B: $\pi^c_{t-1} \geq \pi_{t-1} \geq -\rho$.

5.1.1 Case 1A: $\pi_{t-1} \geq \pi^c_{t-1} > -\rho$

Equation (10) implies that in this case the zero lower bound on the nominal interest rate is not binding. Therefore, equation (8) gives us the period-$t$ inflation rate, $\pi_t$. By Proposition 3, if there are no further shocks from period $t+1$ onwards, we need $\pi_t > -\rho$ to avoid a deflationary spiral (from period $t+1$ onwards). It is clear from equation (8) that a large enough and negative inflation shock and/or demand shock (from falling private-sector confidence, for example) could lead to $\pi_t < -\rho$ (and, therefore, a deflationary spiral). However, equation (8) also makes clear that this danger can be neutralized by expansionary fiscal policy ($\epsilon_t \uparrow$) and/or an increase in the central bank’s target inflation ($\pi^* \uparrow$). (As is clear from equation (8), the effects of $\theta_{\pi t}$ and $\theta_{Y t}$, which represents the responsiveness of the central bank’s interest-setting rule to inflation and output, respectively, on inflation are ambiguous.)

These issues can be explored graphically. Recall that $ABC$ in Figure 2 is the inflation mapping curve for the case without shocks. In Figures 3 and 4, a reduction in private-sector demand ($\epsilon_t \downarrow$) lowers the inflation mapping curve from $ABC$ to $A^1B^1C^1$, thereby threatening to reduce inflation below $-\rho$ and initiate a deflationary spiral. However, Figures 3 and 4 show that with a timely increase in the inflation target ($\pi^* \uparrow$) or fiscal
stimulus \((\epsilon_t \uparrow)\), we can avoid a deflationary spiral.

5.1.2 Case 1B: \(\pi_{t-1}^c \geq \pi_{t-1} \geq -\rho\)

Equation (10) implies that in this case the zero lower bound on the nominal interest rate is binding. Therefore, \(\pi_t\) is now obtained from equation (15). As in Case 1A above, a large enough and negative inflation shock and/or demand shock can initiate a deflationary spiral from period \(t + 1\) onwards. However, as equation (15) makes clear, expansionary fiscal policy \((\epsilon_t \uparrow)\) can neutralize the threat of a deflationary spiral, as was the case in Case 1A. Note that, in contrast to Case 1A, changes in \(\pi^*\) are now of no help as a preventive strategy. This is because monetary policy cannot affect an economy that is at the zero lower bound.

To summarize, raising the inflation target can keep help us avoid a deflationary spiral, but only when such a spiral is already unlikely (Case 1A: \(\pi_{t-1} \geq \pi_{t-1}^c > -\rho\)). Expansionary fiscal policy, on the other hand, works as a preventive in all cases.

5.2 Getting Out of a Deflationary Spiral

Assume \(\pi_{t-1} < -\rho\). Then, by Proposition 3 and assuming no further shocks from period \(t\) onwards, a deflationary spiral will begin in period \(t\). We can, however, rescue this economy by doing something in period \(t\) to ensure \(\pi_t > -\rho\). As the discussion immediately following equation (11) makes clear, we have \(\pi_{t-1} < -\rho < \pi_{t-1}^c\), which implies that the zero lower bound on the nominal interest rate is binding \((\iota_t = 0)\). Therefore, we can seek guidance from equation (15). It is clear from equation (15) that the only way out is expansionary fiscal policy \((\epsilon_t \uparrow)\). As we saw before, monetary policy is useless when the nominal interest rate is stuck at zero.

Figure 5 demonstrates the role of fiscal stimulus in rescuing an economy from a deflationary spiral.

To summarize our discussion of policy responses to a deflationary spiral, we state the following:

**Proposition 4.** A deflationary spiral can be prevented by raising the central bank’s target inflation rate, but only when the previous period’s inflation is high enough. Other mon-
Figure 3: **Monetary Policy to Avoid a Deflationary Spiral** Suppose a decrease in private-sector confidence lowers the inflation mapping curve from $ABC$ to $A^1B^1C^1$, and suppose $\pi_{t-1} = \pi^*$ as in the orthodox long-run equilibrium and is denoted by point $D$. In this case, $\pi_t$ would fall below $-\rho$ and a deflationary spiral would begin, unless preventive measures are taken. An increase in the central bank’s inflation target, $\pi^*$, that raises the inflation mapping curve to $A^1B^2C^2$ (or higher) averts the spiral.
Figure 4: **Fiscal Policy to Avoid a Deflationary Spiral** Suppose a decrease in private-sector confidence lowers the inflation mapping curve from $ABC$ to $A^1 B^1 C^1$, and suppose $\pi_{t-1} = \pi^*$ as in the orthodox long-run equilibrium and is denoted by point $D$. In this case, $\pi_t$ would fall below $-\rho$ and a deflationary spiral would begin, unless preventive measures are taken. The implementation of fiscal stimulus ($\epsilon_t \uparrow$) that raises the inflation mapping curve to $A^2 B^2 C^2$ (or higher) averts the spiral.
Figure 5: Fiscal Policy to Recover from a Deflationary Spiral Suppose the mapping from $\pi_{t-1}$ to $\pi_t$ is represented by $ABC$ and $\pi_{t-1}$ is given by point $D$ and, therefore, is less than $-\rho$. Therefore, a deflationary spiral has already begun. However, a fiscal stimulus ($\epsilon_t \uparrow$) that raises the inflation mapping curve to $A^1B^1C^1$ (or higher) pushes $\pi_t$ above $-\rho$, thereby stopping the spiral.
etary policy tools have an ambiguous preventive role. Expansionary fiscal policy can be used for both prevention and cure.

6 Simulations

In this section, we present some numerical simulations of the model for the parameter values in Table 1.

Table 1: Parameter Values

<table>
<thead>
<tr>
<th>$\bar{Y}_t$</th>
<th>$\alpha$</th>
<th>$\rho$</th>
<th>$\phi$</th>
<th>$\theta_\pi$</th>
<th>$\theta_Y$</th>
<th>$\pi^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.0</td>
<td>2.0</td>
<td>0.25</td>
<td>0.5</td>
<td>0.5</td>
<td>2.0</td>
</tr>
</tbody>
</table>

The first three parameters ($\bar{Y}_t$, $\alpha$, $\rho$) appear in the aggregate demand equation (1). A value of 100 for the natural level of output is convenient as fluctuations in $Y_t - \bar{Y}_t$ can be interpreted as percentage deviation of output from its natural level. The parameter $\alpha = 1$ implies that a one percentage point increase in the real interest rate reduces the output gap by one percentage point. The natural rate of interest $\rho$ is equal to 2 percent, in line with historical data. The parameter $\phi$ appears in the Philips curve in equation (3). The value of 0.25 implies that inflation rises by 0.25 percentage point when output is one percent above its natural level. Finally, the last three parameters affect the Taylor rule in equation (5). The value of one half for the coefficients $\theta_\pi$ and $\theta_Y$ stems directly from Taylor (1993). For each percentage point that inflation rises above the Fed’s inflation target, the federal funds rate rises by 0.5 percent. For each percentage point that real GDP rises above the natural level of output, the federal funds rate rises by 0.5 percent. Since $\theta_\pi$ is greater than zero, our equation for monetary policy follow the Taylor principle: whenever inflation increases, the central bank raises the nominal interest rate by an even larger amount\(^8\). Finally, the inflation target $\pi^*$ is equal to 2 percent, reflecting the Fed’s

\(^8\)Clarida et al. (2000) examine data on inflation, interest rates, and output and estimate the coefficients of the Taylor rule at different points in time. During the Volcker-Greenspan area, they find that $\theta_\pi = 0.72$, implying that the Taylor rule under these two chairmen satisfied the Taylor principle. On the other hand, during the pre-Volcker era from 1960 to 1978, they find that $\theta_\pi = -0.14$. The negative value for $\theta_\pi$
concerns of keeping inflation under control, a policy that started thirty years ago under chairman Paul Volcker.

We present below the paths for output, inflation, nominal and real interest rates when the economy is hit by a four-period contractionary demand shock. Given our choice of parameter values, it takes an output loss of at least 24 percent over four consecutive periods to trigger a deflation-induced depression. The likelihood of such large shocks is infinitesimal. To put things into perspective, output in the U.S. during the Great Depression contracted by 46 percent between 1929 and 1932.

We also ask how aggressive should the policy response be to prevent or cure a depression? Everything else equal, a modest rise in the central bank’s inflation target from 2 to 3 percent is enough to prevent the start of a deflationary spiral and only a fiscal stimulus packages of 4 percent of real output has a chance to get the economy out the deflationary spiral, when it is already in one.

7 Conclusion

Several of today’s leading textbooks for intermediate macroeconomics courses include a dynamic 3-equation New Keynesian model of short-run macroeconomics consisting of an IS curve, a Phillips curve, and a monetary policy rule. In this paper, we have shown that when the DAD-DAS model in Mankiw (2010) is generalized to incorporate the zero lower bound on the nominal interest rate, it has two long-run equilibria, one stable and the other unstable. We have demonstrated the existence of a deflationary spiral in which both output and inflation fall without bound. We have also described policy responses that can keep an economy out of the deflationary spiral and/or rescue it from such a spiral in case one has already begun.

We realize that a deflationary spiral in which output falls without bound is unrealistic. The implicit assumption of wage rigidity in this model would be hard to justify in an

means that the Taylor rule did not follow the Taylor principle. The Fed raised interest rates in response to rising inflation but not enough, implying that real interest rates fell. The authors believe that the wrong specification of the Taylor rule led to the Great Inflation of the 1970s and propose some tentative explanations for why the Fed was so passive in that earlier era.
economy suffering from very high unemployment. At very high rates of unemployment wages would likely drop and, consequently, businesses would likely begin hiring again as the economy’s self-correcting properties kick in. Adapting this paper’s model to deal comprehensively with the deflationary spiral remains a topic for future research.

Monetary policy in the United States and Japan—to take just two examples—has been stuck at the zero lower bound since 2008 and 1995, respectively. Students need to see how short-run macroeconomics works under these new—but no longer unusual—circumstances.
References


8 Appendix: Derivation of Equilibrium Outcomes

8.1 Output and inflation when the zero lower bound is not binding; equations (7) and (8)

Our goal here is to derive expressions for output and inflation as a function of the pre-determined inflation in period $t - 1$ and the model’s “deep” parameters, when the zero lower bound on the nominal interest rate is not binding.

Our first step is to use the Fisher equation (2) to replace the real interest rate in equation (1) by $r_t = i_t - \pi_t$. We get the expression for nominal interest rate from the Taylor rule in equation (5) and obtain the following relationship between $\pi_t$ and $Y_t$:

$$Y_t = \bar{Y}_t - \alpha(\theta_\pi(\pi_t - \pi^*_t) + \theta_Y(Y_t - \bar{Y}_t)) + \epsilon_t$$ (17)

Next, we replace $\pi_t$ in the previous equation by using the Phillips curve (3):

$$Y_t = \bar{Y}_t - \alpha(\theta_\pi(\pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t - \pi^*_t) + \theta_Y(Y_t - \bar{Y}_t)) + \epsilon_t$$ (18)

Note that, in the previous expression, output is the only endogenous variable at time $t$. As a result, we can solve for $Y_t$ as a function of the pre-determined inflation in period $t - 1$ and the model parameters. After collecting the terms in $Y_t$, we obtain:

$$Y_t = \bar{Y}_t + \frac{\alpha \theta_\pi(\pi^*_t - \pi_{t-1} - \nu_t) + \epsilon_t}{1 + \alpha(\theta_\pi\phi + \theta_Y)}$$ (19)

which is equation (7).

Note that the output gap can be obtained from the previous expression:

$$Y_t - \bar{Y}_t = \frac{\alpha \theta_\pi(\pi^*_t - \pi_{t-1} - \nu_t) + \epsilon_t}{1 + \alpha(\theta_\pi\phi + \theta_Y)}$$ (20)

To obtain an expression for inflation in period $t$, we substitute the output gap into the Phillips curve (3) and use our assumption that agents have adaptive expectations. That is, $E_{t-1}\pi_t = \pi_{t-1}$. After collecting terms in $\pi_t$, we obtain the following expression for $\pi_t$ in terms of the parameters of the model and $\pi_{t-1}$:

$$\pi_t = \pi_{t-1} + \frac{\alpha \theta_\pi\phi(\pi^*_t - \pi_{t-1} - \nu_t) + \phi \epsilon_t}{1 + \alpha(\theta_\pi\phi + \theta_Y)} + \nu_t,$$ (21)

which, when rearranged, yields equation (8).
8.2 Nominal interest rate when the zero lower bound is not binding—equation (9)

The equilibrium interest rate in (9) is obtained by substituting the expression for output gap and inflation into the Taylor rule. After some tedious algebra that involve collecting terms and rearranging, we obtain the following expression for interest rate:

\[ i_t = \rho + \left(1 + \theta \pi_t + \alpha \theta Y_t\right)(\pi_{t-1} + \nu_t) - \frac{(1 - \alpha \phi)\theta_n \pi_t^* + (\theta Y_t + (1 + \theta_n)\phi)\epsilon_t}{1 + \alpha \cdot (\theta_n \phi + \theta Y_t)}, \tag{22} \]

which is equation (9).

To solve for the inflation threshold, set \( i_t = 0 \) in the previous expression and solve for the value of \( \pi_{t-1} \). After tedious rearranging of terms, we obtain:

\[ \pi_{t-1}^c = \frac{(1 - \alpha \phi)\theta_n \pi_t^* - (1 + \alpha \theta_n \phi + \alpha \theta Y_t)\rho - (\theta_n \phi + \theta Y_t + \phi)\epsilon_t}{1 + \theta_n + \alpha \theta Y_t} - \nu_t, \tag{23} \]

which is equation (11).

8.3 Output and inflation when the zero lower bound is binding—equations (14) and (15)

Our goal here is to derive expressions for output and inflation as a function of the pre-determined inflation in period \( t - 1 \) and the model’s parameters, when the zero lower bound on the nominal interest rate is binding.

As before, our first step is to use the Fisher equation (2) to replace the real interest rate in equation (1) by \( r_t = i_t - \pi_t \). However, since \( i_t = 0 \), aggregate demand when the zero lower bound is binding is given by:

\[ Y_t = \bar{Y}_t + \alpha(\pi_t + \rho) + \epsilon_t \tag{24} \]

Next, we replace \( \pi_t \) in the previous equation by using the Phillips curve (3):

\[ Y_t = \bar{Y}_t - \alpha(\pi_{t-1} + \phi(\bar{Y}_t - \bar{Y}_t) + \nu_t + \rho) + \epsilon_t \tag{25} \]

We solve for \( Y_t \) as a function of the pre-determined inflation level \( \pi_{t-1} \) and the model parameters:

\[ Y_t = \bar{Y}_t + \frac{\alpha(\pi_{t-1} + \nu_t + \rho) + \epsilon_t}{1 - \alpha \phi}, \tag{26} \]
which is equation (14).

Note that the output gap can be obtained from the previous expression:

\[ Y_t - \bar{Y}_t = \frac{\alpha(\pi_{t-1} + \nu_t + \rho) + \epsilon_t}{1 - \alpha \phi}. \]  \hfill (27)

To obtain an expression for inflation in period \( t \), we substitute the output gap into the Phillips curve (3) and use our assumption that agents have adaptive expectations. That is, \( E_{t-1}\pi_t = \pi_{t-1} \). After collecting terms in \( \pi_t \), we obtain the following expression for \( \pi_t \) in terms of the parameters of the model and \( \pi_{t-1} \):

\[ \pi_t = \pi_{t-1} + \frac{\alpha \phi(\pi_{t-1} + \nu_t + \rho) + \phi \epsilon_t}{1 - \alpha \phi} + \nu_t. \]  \hfill (28)

After further simplification, we get

\[ \pi_t = \frac{1}{1 - \alpha \phi}(\pi_{t-1} + \nu_t) + \frac{\phi}{1 - \alpha \phi}(\alpha \rho + \epsilon_t), \]  \hfill (29)

which is equation (15).