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A Simple Treatment of the Liquidity Trap for Intermediate Macroeconomics Courses

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CONTENT ARTICLE IN ECONOMICS

A Simple Treatment of the Liquidity Trap for Intermediate Macroeconomics Courses

Sebastien Buttet and Udayan Roy

Several leading undergraduate intermediate macroeconomics textbooks now include a simple reduced-form New Keynesian model of short-run dynamics (alongside the IS-LM model). Unfortunately, there is no accompanying description of how the zero lower bound on nominal interest rates affects the model. In this article, the authors show how the aforementioned model can easily be modified to teach undergraduate students about the significance of the zero lower bound for economic performance and policy. This acquires additional significance because economies such as the United States and Japan have been close to the zero lower bound since 2008 and 1995, respectively. The authors show that when the zero lower bound is introduced, an additional long-run equilibrium exists. This equilibrium is unstable and can lead to a deflationary spiral.

Keywords *fiscal policy, intermediate macroeconomics, monetary policy, zero lower bound*

JEL codes *E12, E52, E62*

The Great Recession of 2008–9 has changed the practice and teaching of macroeconomics. Authors of prominent textbooks have added entire chapters on the crisis and its aftermath in their new editions. The economy-wide ripples of developments in the financial sector have received considerable coverage. The zero lower bound (ZLB) on nominal interest rates is no longer ignored or treated as a curiosity. In this article, we demonstrate a simple way to improve the integration of the ZLB into the discussion of short-run macroeconomic theory and policy in undergraduate macroeconomics textbooks.

Even a quick look at recent editions of prominent undergraduate macroeconomics textbooks such as Mankiw (2013) or Jones (2011) shows that the IS-LM model is being supplemented by a reduced-form New Keynesian model consisting of the IS curve, the expectations-augmented Phillips curve, and a monetary policy rule for the central bank. Unfortunately, in these textbook

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discussions, the central bank's monetary policy rule simply ignores the ZLB; it is often mentioned, but it is not built into the theory. This will not do in a world in which the central banks of the United States and Japan have kept their policy rates near zero since 2008 and 1995, respectively. Bright and curious students will inevitably ask their instructors how the graphs of the New Keynesian model that they have been taught would look when the central bank is at the ZLB. In this article, we show that the integration of the ZLB into the New Keynesian model is remarkably straightforward and can yield interesting insights into, for instance, the dreaded phenomenon of the deflationary spiral.

In several of today's undergraduate macroeconomics textbooks, the dynamic properties of New Keynesian models are analyzed with two curves—a negatively sloped aggregate demand curve and a positively sloped aggregate supply curve—that link inflation and output. The intersection of these two curves determines equilibrium output and inflation. We add the explicit requirement that the nominal interest rate set by the central bank must be non-negative. We show that the familiar negatively sloped aggregate demand curve becomes a kinked curve with a negatively sloped segment (when the ZLB is nonbinding) and a positively sloped segment (when the ZLB is binding). The positively sloped segment captures the idea that falling inflation is a special nightmare at the ZLB. As nominal interest rates cannot be reduced any further, any decline in inflation means an increase in the real interest rate which in turn reduces aggregate demand and output.

The kinked demand curve generates two (rather than one) long-run equilibria: (1) a stable equilibrium where nominal interest rates are positive and inflation is equal to the central bank's target rate of inflation and (2) an unstable equilibrium at which the nominal interest rate is zero and even the slightest shock can set off a deflationary spiral.

We show that the convergence properties of the economy depend on whether or not inflation is less than a tipping-point rate. As long as the inflation rate exceeds the negative of the natural (or long-run) real interest rate, the economy converges to the stable long-run equilibrium—even if the ZLB is initially binding. On the other hand, if inflation falls below the negative of the natural real interest rate, the economy enters a deflationary spiral with continuously falling inflation and output.

With regard to the effectiveness of monetary and fiscal policy in dealing with the deflationary spiral, we ask two questions: (1) What can be done to keep an economy away from the deflationary spiral? and (2) Can policy get an economy out of a deflationary spiral if it is already in one? We show that expansionary fiscal policy is an adequate answer to both questions, while expansionary monetary policy—specifically, an increase in the target inflation rate—is a partial answer to (1) only.

The main ideas within our article—(1) that the ZLB introduces a new long-run equilibrium; (2) that that equilibrium is unstable; (3) that the ZLB introduces a deflationary spiral; and (4) that there is a tipping point beyond which deflation leads to the deflationary spiral—have been explained here in the graphical form of chapter 15 in Mankiw (2013). All that an instructor who teaches that chapter would have to do is show (a) that the central bank's monetary policy rule does not always yield a positive nominal interest rate, and (b) that when the monetary policy rule yields a negative nominal interest rate, the ZLB kicks in and the aggregate demand curve becomes positively sloped. Our four main results then immediately follow.

It is clear to us that unless the investment required—from both teacher and student—for a discussion of the ZLB is kept low, our treatment of the ZLB would be unlikely to be useful in the classroom. This is why we have expressed our analysis in the graphical style of Mankiw's chapter 15. We have shown that the main results that follow from the introduction of the ZLB can be taught to undergraduates with a simple change to Mankiw's (dynamic) aggregate demand curve. We believe that use of our article's contents sharply reduces the marginal cost of teaching the ZLB. Also, given that the ZLB yields several interesting results, the marginal benefit of teaching Mankiw's chapter 15 (or its counterpart in other textbooks) may now be higher for those instructors who currently skip the chapter.

Although our analysis builds on Mankiw (2013), to make our analysis relevant and useful for instructors who do not use that textbook, our penultimate section reviews the treatment of the ZLB in five other prominent intermediate macroeconomics textbooks: Blanchard and Johnson (2013), Carlin and Soskice (2006), Jones (2011), Gordon (2012), and Mishkin (2011). Several of these textbook authors present algebraic-cum-graphical models in which the intersection of (aggregate) demand and supply curves determine output and inflation. However, none of them show how the ZLB affects their graphs. We explain how instructors who teach the New Keynesian model can modify their graphs and teach the ZLB with minimal hassle.

The remainder of the article is organized as follows: In the next two sections, we introduce our model and characterize its long-run equilibria. We discuss the stability of these equilibria in our fourth section, and we study policy responses to a deflationary environment in our fifth section. We discuss the treatment of the ZLB in several notable intermediate macroeconomics textbooks in our penultimate section. We offer brief concluding remarks in our last section.

A MODEL OF SHORT-RUN MACROECONOMIC DYNAMICS

Our goal here is to take a typical model of short-run macroeconomic dynamics from a standard undergraduate intermediate macroeconomics textbook and demonstrate how the analysis can be enriched if a non-negativity constraint on the nominal interest rate is added. The model that we have chosen to use is variously referred to as the dynamic AD-AS (or DAD-DAS) model in Mankiw (2013, ch. 15), the AS/AD model in Jones (2011), and the three-equation (IS-PC-MR) model in Carlin and Soskice (2006), where PC refers to the Phillips curve and MR refers to the central bank's monetary policy rule. This dynamic model has begun to supplement the static IS-LM model as the mainstay of short-run analysis in undergraduate macroeconomics textbooks. It is our belief that adding the ZLB to the teaching of short-run macroeconomic dynamics in undergraduate courses will increase the realism and relevance of the analysis because the interest rates used by central banks as their instruments of monetary policy have been close to zero for a long time in several countries. For example, the bank rate of the Bank of England has been at 0.5 percent since 2009. The federal funds rate, which is the policy rate for the Federal Reserve in the United States, has been near zero since October 2008. The official discount rate in Japan has been close to zero since 1995.

For specificity, we look at how the ZLB affects the DAD-DAS model in Mankiw (2013, ch. 15). We begin by examinimg the five equations that drive Mankiw's DAD-DAS model. Equilibrium in the market for goods and services is given by

$$Y_t = \bar{Y}_t - \alpha \cdot (r_t - \rho) + \varepsilon_t \tag{1}$$

where Y_t is real output at time t, $\bar{Y}_t$ is the natural or long-run level of output, r_t is the real interest rate, ρ is the natural or long-run real interest rate, α is a positive parameter representing the responsiveness of aggregate expenditure to the real interest rate, and ε_t represents demand shocks.[1] This equation is essentially the well-known IS curve of the IS-LM model, and it has no intertemporal dynamics. The shock ε_t represents exogenous shifts in demand that arise from changes in consumer and/or business sentiment—the so-called "animal spirits"—as well as changes in fiscal policy. When the government implements a fiscal stimulus (an increase in government expenditure or a decrease in taxes), ε_t is positive, whereas fiscal austerity makes ε_t negative.

The ex ante real interest rate in period t is determined by the Fisher equation (1933) and is equal to the nominal interest rate i_t minus the inflation expected currently for the next period:

$$r_t = i_t - E_t\pi_{t+1}. \tag{2}$$

Inflation in the current period, π_t, is determined by a conventional Phillips curve augmented to include the role of expected inflation, $E_{t-1}\pi_t$, and exogenous supply shocks, ν_t:

$$\pi_t = E_{t-1}\pi_t + \phi \cdot \left(Y_t - \bar{Y}_t\right) + \nu_t \tag{3}$$

where ϕ is a positive parameter.

Inflation expectations play a key role in both the Fisher equation (2) and the Phillips curve (3). As in Mankiw, we assume that inflation in the current period is the best forecast for inflation in the next period. That is, agents have adaptive expectations:

$$E_t\pi_{t+1} = \pi_t. \tag{4}$$

We complete the description of the DAD-DAS model with a monetary policy rule. Dynamic New Keynesian models assume that the central bank sets a target for the nominal interest rate, i_t, based on the inflation gap and the output gap, as in the Taylor rule (Taylor 1993). Specifically, the DAD-DAS model in Mankiw assumes that the monetary policy rule is $i_t = \pi_t + \rho + \theta_\pi \cdot (\pi_t - \pi^*) + \theta_Y \cdot (Y_t - \bar{Y}_t)$, where the central bank's inflation target (π^*) and its policy parameters θ_π and θ_Y are all non-negative. We, however, wish to explicitly incorporate the fact that nominal interest rates must be non-negative. Therefore, our generalized monetary policy rule is

$$i_t = \max\left\{0, \pi_t + \rho + \theta_\pi \cdot \left(\pi_t - \pi^*\right) + \theta_Y \cdot \left(Y_t - \bar{Y}_t\right)\right\}. \tag{5}$$

Equations (1) through (5) describe our DAD-DAS model. For given values of the model's period-t parameters (α, ρ, ϕ, θ_π, θ_Y, π^*, and $\bar{Y}_t$), its period-t shocks (ε_t and ν_t), and the predetermined inflation rate ($E_{t-1}\pi_t = \pi_{t-1}$) for period $t-1$, one can use the model's five equations to solve for its five period-t endogenous variables (Y_t, r_t, i_t, $E_t\pi_{t+1}$, and π_t). Once it is understood that the inherited inflation rate (π_{t-1}, which is also the previous period's equilibrium inflation rate) determines the current equilibrium inflation rate, one sees the dynamics that are internal to the DAD-DAS model. Parameter changes and/or shocks are not the only source of change; what happened yesterday determines what happens today which determines what happens tomorrow, and so on.[2]

For the graphical treatment of the model, we will—following Mankiw—turn our five equations that contain five endogenous variables into two equations that contain two endogenous variables, Y_t and π_t. The two equations will then be graphed as the dynamic aggregate demand (DAD) and dynamic aggregate supply (DAS) curves, with Y_t on the horizontal axis and π_t on the vertical axis. The intersection of the two curves will determine the equilibrium values of Y_t and π_t.

The Kinked DAD Curve

Here, we introduce the *only* element of the DAD-DAS model of Mankiw that changes when the ZLB on the nominal interest rate is added. To give away the punchline, Mankiw's negatively sloped DAD curve becomes a kinked DAD curve with a negatively sloped segment (when the ZLB is nonbinding) and a positively sloped segment (when the ZLB is binding).

Figure 1 shows, among other things, the border that separates the (Y_t, π_t) outcomes for which the ZLB *is not* binding from the (Y_t, π_t) outcomes for which the ZLB *is* binding. Algebraically, the monetary policy rule (5) implies that this border satisfies

$$\pi_t + \rho + \theta_\pi \cdot \left(\pi_t - \pi^*\right) + \theta_Y \cdot \left(Y_t - \bar{Y}_t\right) = 0. \tag{6}$$

Above this border, the ZLB is not binding and the nominal interest rate set by the central bank is positive ($i_t > 0$); Mankiw's analysis applies to this case word for word. On the border, the central bank chooses a zero interest rate, but does so willingly, and not because it wanted a negative rate but could not choose it because of the ZLB. Below the border, the ZLB is binding ($i_t = 0$).[3]

Mankiw derives the equation of his DAD curve as follows: Substitute adaptive expectations (4) and Mankiw's simplified monetary policy rule $[i_t = \pi_t + \rho + \theta_\pi \cdot (\pi_t - \pi^*) + \theta_Y \cdot (Y_t - \bar{Y}_t)]$

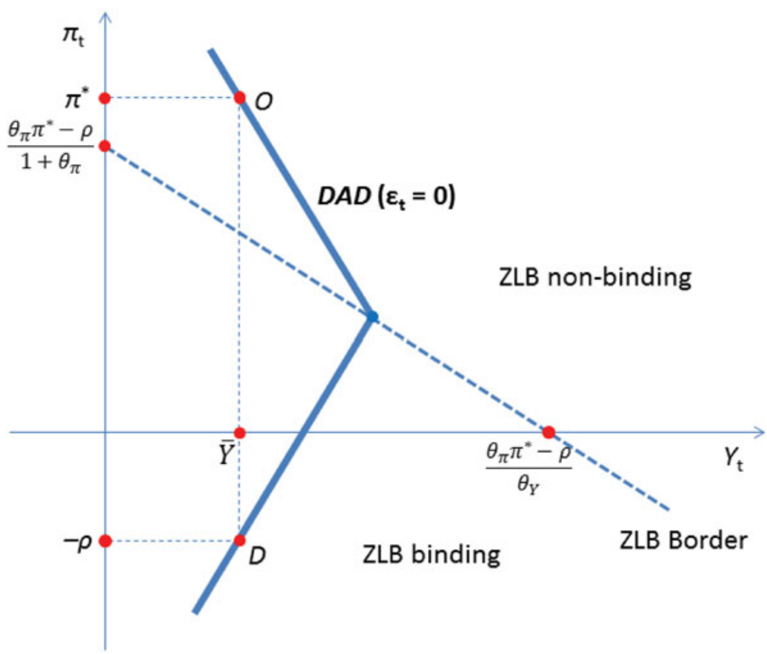

FIGURE 1 Kinked DAD curve is shown here. It is assumed that $\varepsilon_t = 0$ and $\bar{Y}_t = \bar{Y}$ for all t (color figure available online).

into the Fisher equation (2), substitute the resulting expression for the real interest rate in the IS equation (1), and rearrange and collect the terms. In this way, Mankiw gets

$$Y_t = \bar{Y}_t - \frac{\alpha\theta_\pi}{1+\alpha\theta_Y}\left(\pi_t - \pi^*\right) + \frac{1}{1+\alpha\theta_Y}\varepsilon_t. \tag{7}$$

This is graphed as the negatively sloped line in the section of figure 1 where the ZLB is not binding; note that $d\pi_t/dY_t < 0$ in equation (7).

The DAD curve in figure 1 assumes that the demand shock is absent ($\varepsilon_t = 0$). Consequently, $\pi_t = \pi^*$ and $Y_t = \bar{Y}_t$ satisfies equation (7). This is point O in figure 1; it will play an important role in our discussion of the model's equilibrium below.

Note that equation (7) implies that Mankiw's DAD curve (i.e., the DAD curve when the ZLB is not binding) shifts rightward under both expansionary monetary policy ($\pi^*\uparrow$) and expansionary fiscal policy ($\varepsilon_t\uparrow$). This is shown in figures 2 and 3.

Repeating Mankiw's procedure, but with $i_t = 0$, we get the DAD curve for the case in which the ZLB is binding:

$$Y_t = \bar{Y}_t + \alpha \cdot (\pi_t + \rho) + \varepsilon_t. \tag{8}$$

Note that the slope is positive ($d\pi_t/dY_t = 1/\alpha > 0$), which is why the DAD curve in figure 1 turns into a positively sloped line below the ZLB border. The familiar negatively sloped DAD

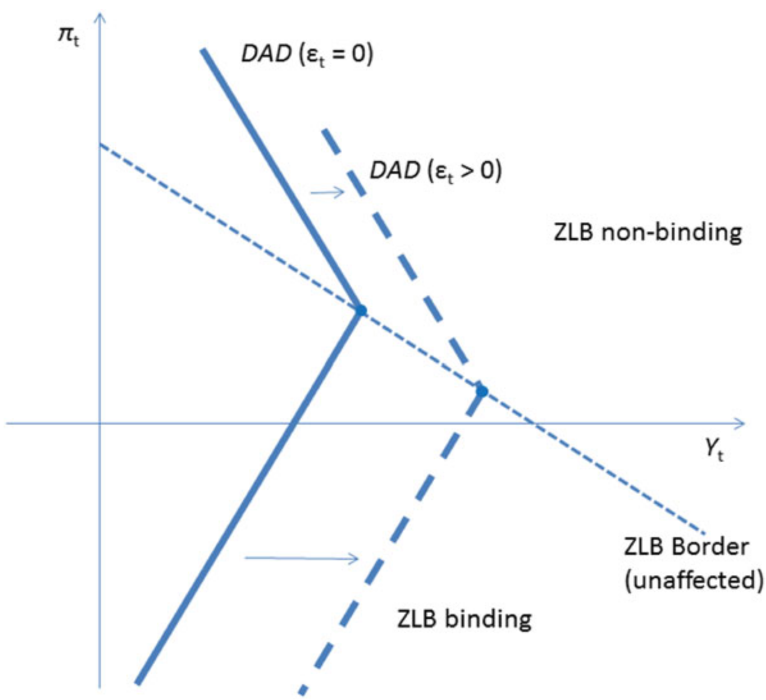

FIGURE 2 Expansionary fiscal policy shifts the kinked DAD curve right without moving the ZLB border (color figure available online).

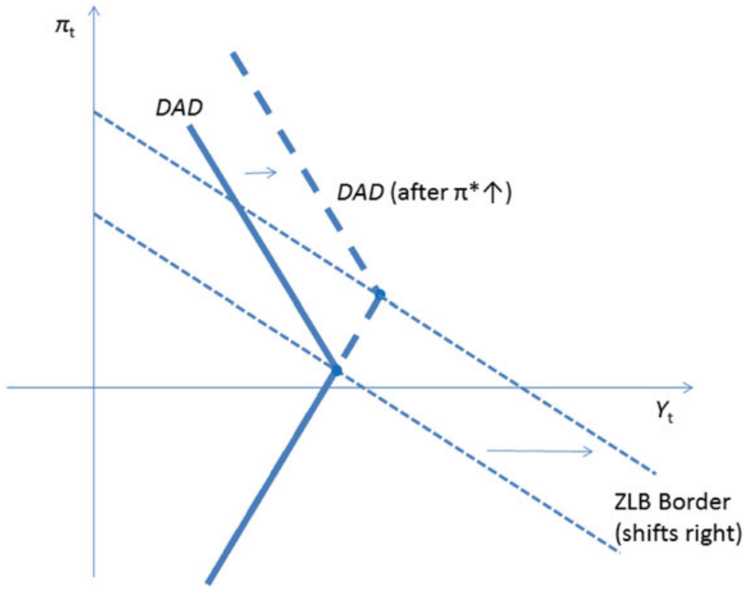

FIGURE 3 Expansionary monetary policy shifts the ZLB border and the part of the DAD above the ZLB border to the right. The part of the DAD below the ZLB border just gets extended (color figure available online).

curve we see in textbooks becomes a kinked curve when the analysis allows the ZLB to be binding. Algebra aside, the positively sloped segment is meant to capture the idea that falling inflation is a special nightmare at the ZLB. As $i_t = 0$, any decline in current inflation means an increase in the current real interest rate: As $r_t = i_t - E_t\pi_{t+1} = 0 - \pi_t = -\pi_t$, we get $dr_t/d\pi_t = -1 < 0$. The rising real interest rate reduces aggregate demand and output, as the familiar IS curve (1) dictates.[4]

By contrast, when the ZLB is not binding, the negatively sloped DAD curve reflects a different story. Any decrease in inflation provokes the monetary policy rule (5) to reduce the nominal interest rate even more, as dictated by the Taylor principle. As a result, the real interest rate *falls*, thereby causing both aggregate demand and output to *increase*.

To finish our discussion of the rising segment of the DAD curve, when $\varepsilon_t = 0$, note that $\pi_t = -\rho$ and $Y_t = \bar{Y}_t$ satisfy equation (8). This is point *D* in figure 1. Like point *O*, *D* too will play an important role in our discussion of the model's equilibrium.

Equation (8) also implies that the positively sloped segment of the DAD curve shifts rightward under expansionary fiscal policy ($\varepsilon_t\uparrow$) and is extended but not shifted by expansionary monetary policy ($\pi^*\uparrow$). This is shown in figures 2 and 3.

The DAS Curve

When adaptive expectations (4) is substituted into the Phillips curve (3), we get Mankiw's DAS curve:

$$\pi_t = \pi_{t-1} + \phi \cdot \left(Y_t - \bar{Y}_t\right) + \nu_t. \tag{9}$$

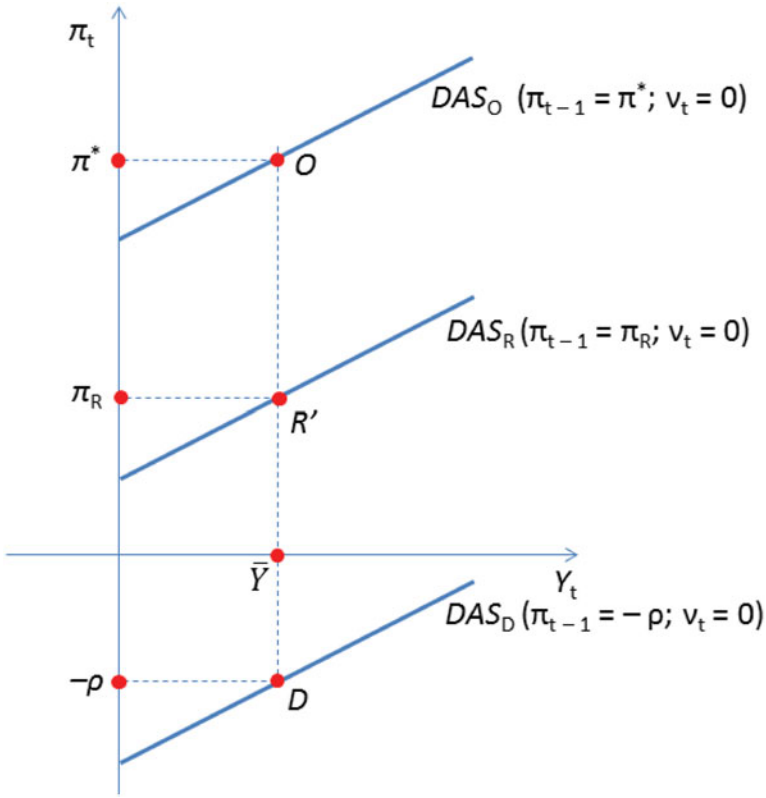

FIGURE 4 DAS curve is shown here. It is assumed that $\nu_t = 0$ and $\bar{Y}_t = \bar{Y}$ for all t (color figure available online).

It follows from equation (9) that the slope of the DAS curve is $d\pi_t/dY_t = \phi > 0$, as shown in figure 4. It also follows that any change in the predetermined inherited rate of inflation (π_{t-1}), or in the supply shock (ν_t), or indeed in $\pi_{t-1} + \nu_t$, leads to an equal change in the height of the DAS curve. The link between π_{t-1} and π_t in the DAS equation (9) is the *only* source of dynamics in the DAD-DAS model; note that the DAD equations (7) and (8) contain only period-t variables.

Finally, note that when the supply shock is assumed absent ($\nu_t = 0$; as in the DAS curves DAS_O, DAS_R, and DAS_D in figure 4), $Y_t = \bar{Y}_t$ and $\pi_t = \pi_{t-1}$ satisfy equation (9). This is the case for the outcomes O, R', and D in figure 4.

Now that we have discussed the DAS and DAD curves, we are ready to bring them together to discuss equilibrium.

LONG-RUN EQUILIBRIA

Equilibrium at any period t is graphically represented by the intersection of the DAD and DAS curves for period t. Figure 5 shows three such equilibria—at O, R, and D—for the same DAD curve and three different DAS curves.

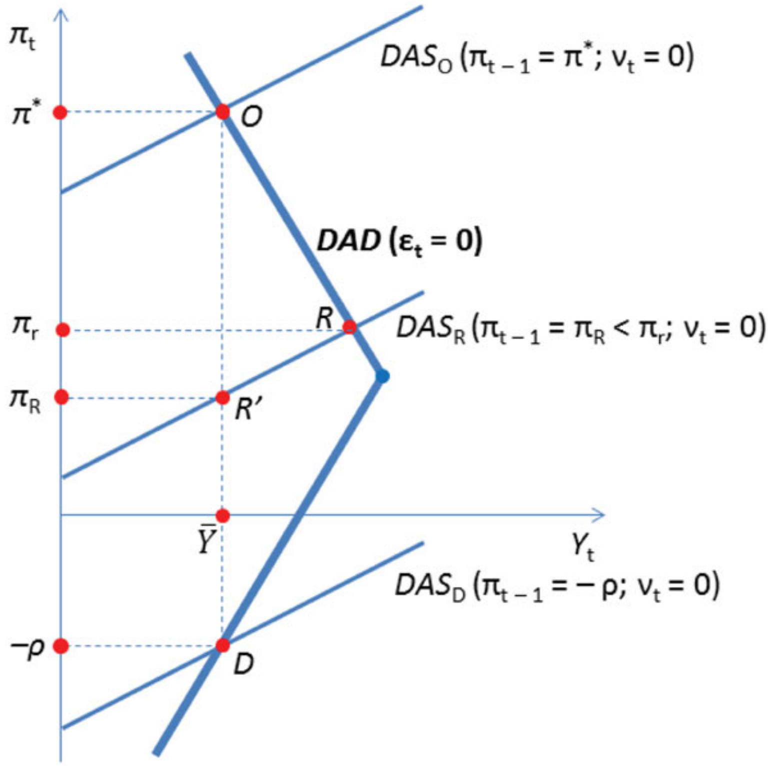

FIGURE 5 Both shocks are assumed zero, and $\bar{Y}_t = \bar{Y}$ for all t (color figure available online).

Let us begin with the equilibrium at R. As can be confirmed by a quick glance at the DAS equation (9), as $Y_t > \bar{Y}$ at R and as $\nu_t = 0$ has been assumed, it must be that $\pi_t > \pi_{t-1}$. That is, inflation is rising over time. Therefore, this is not what Mankiw calls a *long-run equilibrium*, which is an equilibrium outcome that repeats itself (as long as the model's parameters stay unchanged and there are no shocks). As a general matter, if no restrictions are imposed on the DAD-DAS model's parameters and shocks, there is no reason to expect an equilibrium outcome to repeat itself period after period. The question then is the following: Do there exist restrictions on the model's parameters and shocks under which an equilibrium outcome would repeat itself over and over again?

Let us now look at the equilibrium at O in figure 5. In this case, $\pi_t = \pi_{t-1} = \pi^*$ and $Y_t = \bar{Y}$. With no parameter changes and no shocks, the DAD curve at $t + 1$ will be identical to the DAD curve at t, which is the one shown in figure 5. Also, as inherited inflation is the same in periods t and $t + 1$ ($\pi_{t-1} = \pi_t = \pi^*$) and $\nu_t = \nu_{t+1} = 0$ by assumption, the DAS curve at $t + 1$ will be identical to the DAS curve at t, which is DAS_O of figure 5. Therefore, O represents the equilibrium outcome for periods $t + 1$ as well as t. In short, we have found an equilibrium that repeats. We can conclude that (a) if the DAD-DAS model's parameters stay constant, (b) if the two shocks stay at zero ($\varepsilon_t = \nu_t = 0$), and (c) if the inherited inflation happens to be equal to the central bank's target inflation ($\pi_{t-1} = \pi^*$), then the equilibrium outcome will continue unchanged forever.

Mankiw goes on to show, algebraically and graphically, that the equilibrium at O in figure 5—which we will henceforth refer to as the *orthodox equilibrium*—is the one and only long-run equilibrium of his DAD-DAS model (which, recall, makes no mention of the ZLB). To fully describe the orthodox equlibrium, note that the monetary policy rule (5) implies $i_t = \pi^* + \rho$, and the Fisher equation (2) and adaptive expectations (4) imply $r_t = i_t - E_t\pi_{t+1} = i_t - \pi_t = \rho$.

With the introduction of the ZLB, however, we now have a kinked DAD curve with a new positively sloped segment, and it is straightforward to check that outcome D in figure 5, at the intersection of the kinked DAD curve and DAS_D, is also a long-run equilibrium. Although output is $\bar{Y}$ and the real interest rate is ρ, exactly as in Mankiw's orthodox equilibrium, the nominal interest rate is zero—we are at the ZLB, after all—and the inflation rate is $-\rho < 0$. We call D the *deflationary equilibrium*.

Before we move on to our discussion of the stability of our two long-run equilibria, a technical issue must be discussed. Note that DAS_D in figure 5 is drawn flatter than the rising part of the DAD curve. This reflects our assumption that $1/\alpha$, which is the slope of the ZLB section of the DAD curve, exceeds ϕ, the slope of the DAS curve. Equivalently, we assume $1 - \alpha\phi > 0$. We discuss this assumption further in a later section on the slopes of the DAD and DAS curves.

STABILITY OF LONG-RUN EQUILIBRIA

We will now show that not only does the ZLB add a new long-run equilibrium—the deflationary equilibrium—to the DAD-DAS model, but also the deflationary equilibrium is unstable, unlike the orthodox equilibrium, which is stable.

Let us assume that the parameters of the DAD-DAS model (α, ρ, ϕ, θ_π, θ_Y, π^*, and $\bar{Y}$) are constant—and both shocks are at zero—from period t onwards. Under these conditions, we saw in our section on long-run equilibria that if $\pi_{t-1} = \pi^*$, the economy will stay at the othodox equilibrium forever, and if $\pi_{t-1} = -\rho$, the economy will stay at the deflationary equilibrium forever. But what if π_{t-1} is neither π^* nor $-\rho$? For arbitrary values of π_{t-1}, how will the economy behave during periods t and later?

Under our assumption that the parameters (α, ρ, ϕ, θ_π, θ_Y, π^*, and $\bar{Y}$) are constant—and both shocks are at zero—from period t onward, the kinked DAD curve will be the same for all periods t and later (as is clear from equations (7) and (8) in our section on the kinked demand curve). Let this DAD curve be the one shown in figure 6.

Case 1: $\pi_{t-1} > \pi^*$. Let $\pi_{t-1} = \pi_Q > \pi^*$. As $\pi_{t-1} > \pi^*$, the DAS curve at period t, indicated in figure 6 by DAS_Q, must be higher than DAS_O, for which inherited inflation was specified to be $\pi_{t-1} = \pi^*$. As we saw in our section on the DAS curve, the height of DAS_Q at $Y_t = \bar{Y}$ is $\pi_{t-1} = \pi_Q > \pi^*$, as shown in figure 6. The equilibrium at period t is, therefore, at q, with $\pi_{t-1} > \pi_t > \pi^*$. In other words, if inherited inflation exceeds π^*, current inflation will be *lower* than inherited inflation while still remaining higher than π^*. Applying this result recursively while keeping in mind that this period's current inflation is next period's inherited inflation, we see that inflation converges to π^*, and the equilibrium outcome converges to the orthodox equilibrium, O.[5]

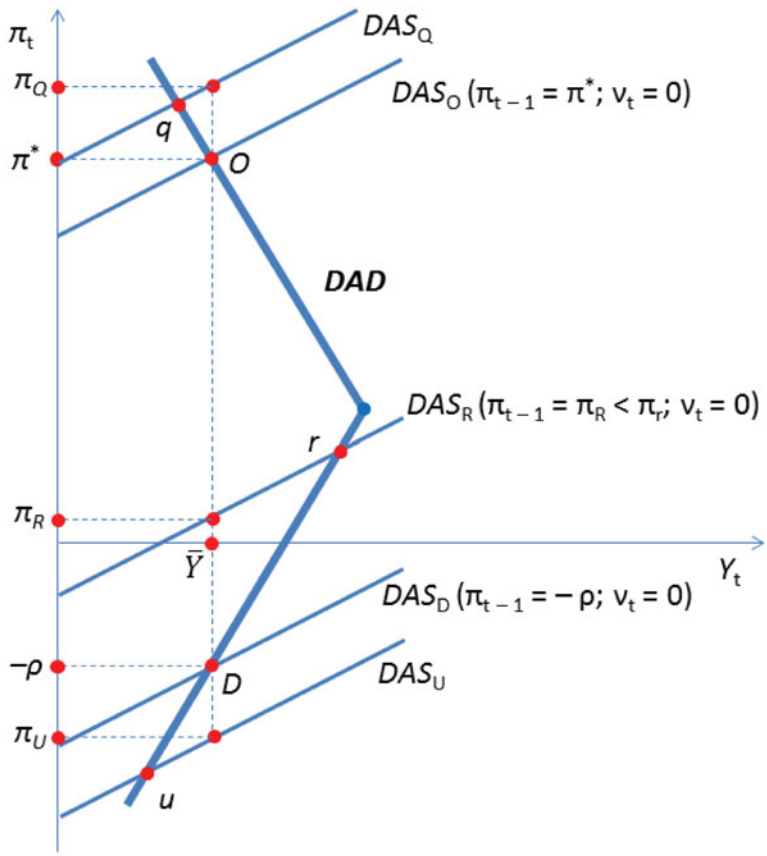

FIGURE 6 The orthodox equilibrium, O, is stable, and the deflationary equilibrium, D, is unstable. $\bar{Y}_t = \bar{Y}$ for all t (color figure available online).

Case 2: $-\rho < \pi_{t-1} < \pi^*$. Let $-\rho < \pi_{t-1} = \pi_{\mathrm{R}} < \pi^*$. Therefore, the DAS curve at t will be somewhere between DAS_O and DAS_D, for which inherited inflation was specified to be $\pi_{t-1} = \pi^*$ and $\pi_{t-1} = -\rho$, respectively. Let this DAS curve be DAS_R in figure 6. The period-t equilibrium is, therefore, at r with $-\rho < \pi_{t-1} < \pi_t < \pi^*$. In other words, if inherited inflation lies between $-\rho$ and π^*, current inflation will be *higher* than inherited inflation while still remaining between $-\rho$ and π^*. Applying this result recursively while keeping in mind that this period's current inflation is next period's inherited inflation, we see that the equilibrium outcome will converge to the orthodox equilibrium, O.

Case 3: $\pi_{t-1} < -\rho$. Let $\pi_{t-1} = \pi_U < -\rho$. The DAS curve at t will be below DAS_D. Let this DAS curve be DAS_U in figure 6. The equilibrium will be at u with $\pi_t < \pi_{t-1} < -\rho$. In other words, if inherited inflation is less than $-\rho$, current inflation will be *lower* than inherited inflation and therefore even farther below $-\rho$. Applying this result recursively while keeping in mind that this period's current inflation is next period's inherited inflation, we see that the equilibrium outcome will diverge from the deflationary equilibrium, D, with both inflation and output falling continuously. This is the much dreaded *deflationary spiral*.[6]

To sum up, we have shown that as long as the parameters of the DAD-DAS model do not change and there are no shocks, the economy will either converge to the orthodox equilibrium or be in the ever-worsening deflationary spiral. The key knife's edge factor is the inflation rate. If inflation falls below $-\rho$, which is the negative of the natural real interest rate, the economy's fate is the deflationary spiral with ever-decreasing inflation and output. If the inflation rate stays above $-\rho$, the economy converges to the orthodox long-run equilibrium at O, and there is no reason to worry.

Recall that in our discussion above we have assumed that ϕ, the slope of the DAS curve, is smaller than $1/\alpha$, the slope of the positively sloped segment of the DAD curve (under the ZLB). We will now argue that this assumption is necessary to avoid comparative static results that seem unrealistic to us.

Consider the equilibrium outcome a at the intersection of DAS and DAD_1 in the left panel of figure 7. Note that contrary to our assumption above, DAS has been drawn steeper than DAD_1. Now consider a positive demand shock ($\varepsilon_t\uparrow$). As we saw in our section on the kinked DAD curve and figure 2, the economy's DAD curve will shift rightward to, say, DAD_1. Therefore, the new equilibrium will be at b. In other words, an *increase* in demand leads to *lower* inflation and *lower* output. This outcome strikes us as unrealistic.

Similarly, in the right panel of figure 7, we see another comparative static result that seems unrealistic to us: an *increase* in the cost shock ($\nu_t\uparrow$), such as increases in the price of imported oil or a series of bad droughts, leads to *lower* inflation.

These unrealistic comparative static results can be avoided by assuming $1/\alpha > \phi$ or, equivalently, $1-\alpha\phi > 0$.

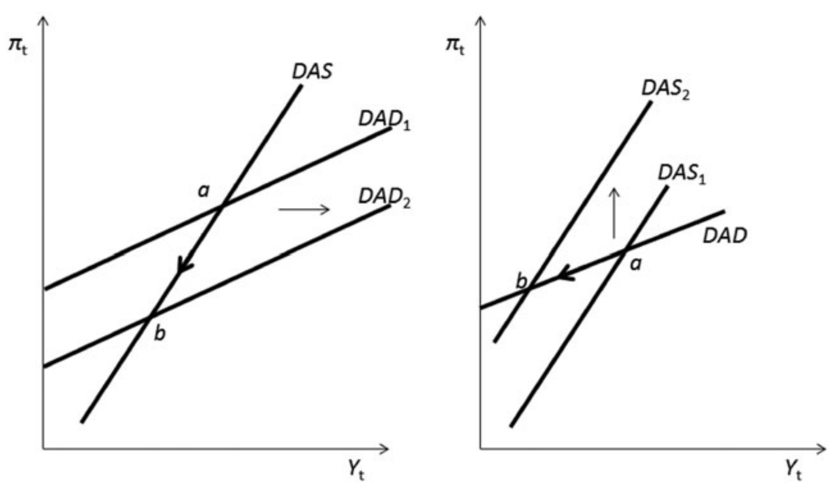

FIGURE 7 If DAS is steeper than DAD, a demand stimulus leads to lower output and lower inflation (left), and a supply shock leads to lower inflation (right).

POLICY RESPONSES TO DEFLATIONARY SPIRALS

Given that a deflationary spiral—with output decreasing without bound—is undesirable, (a) what can be done to keep an economy away from it, and (b) what can be done to get an economy out of a deflationary spiral if it is already in one. We will show that expansionary fiscal policy—that is, an increase in ε_t in the goods market's equilibrium condition (1)—is an adequate answer to both questions, and expansionary monetary policy—that is, an increase in the central bank's target inflation rate (π^*) in the monetary policy rule (5)—is a partial answer to (a).

Fiscal Stimulus Works

Recall that figure 2 shows how expansionary fiscal policy ($\varepsilon_t > 0$) shifts our kinked DAD curve to the right. This is reproduced in figure 8.

Suppose inflation has fallen below $-\rho$, and consequently, the economy has already fallen into a deflationary spiral.[7] Suppose the DAS curve is expected to be at DAS_U in period t. In that case, if there is no government intervention, the period t equilibrium will be at u in figure 8. However, as in the figure, expansionary fiscal policy in period t can move the equilibrium to v, thereby lifting the inflation rate above $-\rho$. Once that is achieved, the fiscal stimulus can be withdrawn (i.e., ε_t

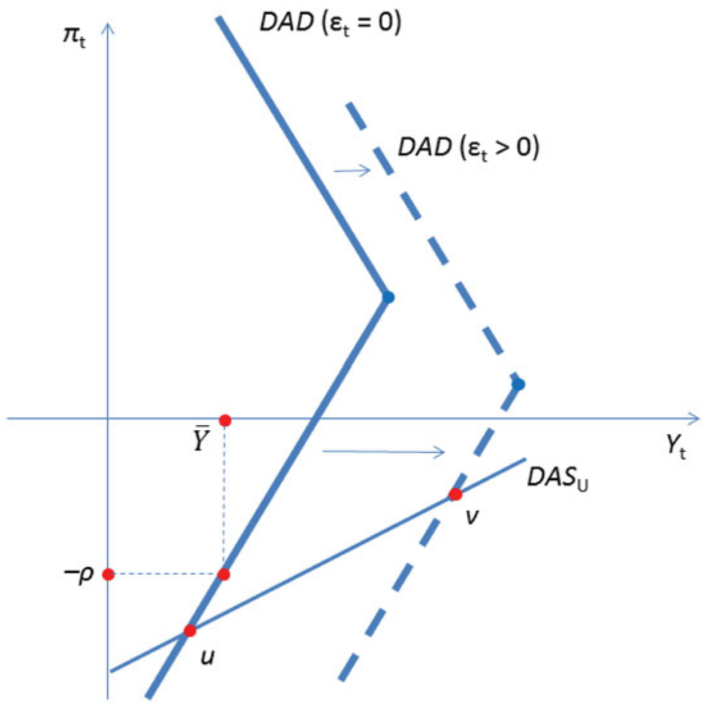

FIGURE 8 Expansionary fiscal policy and the deflationary spiral. $\bar{Y}_t = \bar{Y}$ for all t (color figure available online).

can return to zero and DAD can return to its original position), and as we have seen before, the economy will gradually converge to the orthodox equilibrium.

The same policy can also be used as a prophylactic. If, for some reason, it is imminent that the DAS curve will drop to DAS_U, we can use expansionary fiscal policy to shift the DAD curve to the right, thereby nipping the deflationary spiral in the bud.

Finally, if inflation has been (or soon will be) pushed below $-\rho$ by a leftward shift in the DAD curve (say, by a decline in "animal spirits" or "confidence"), then it goes without saying, expansionary fiscal policy can negate such a leftward shift.

Expansionary Monetary Policy May Work

Recall that figure 3 shows how expansionary monetary policy ($\pi^*\uparrow$) shifts our kinked DAD curve. This is reproduced in figure 9.

Suppose the economy is at u in figure 9, inflation has dropped below $-\rho$, and therefore, a deflationary spiral is already underway. Expansionary monetary policy (which can only *extend* the positively sloped segment of the DAD curve but not *shift* it) is of no use in this case. A deflationary spiral can only occur when the ZLB on the nominal interest rate is binding. As a result, monetary policy is ineffective in a deflationary spiral.

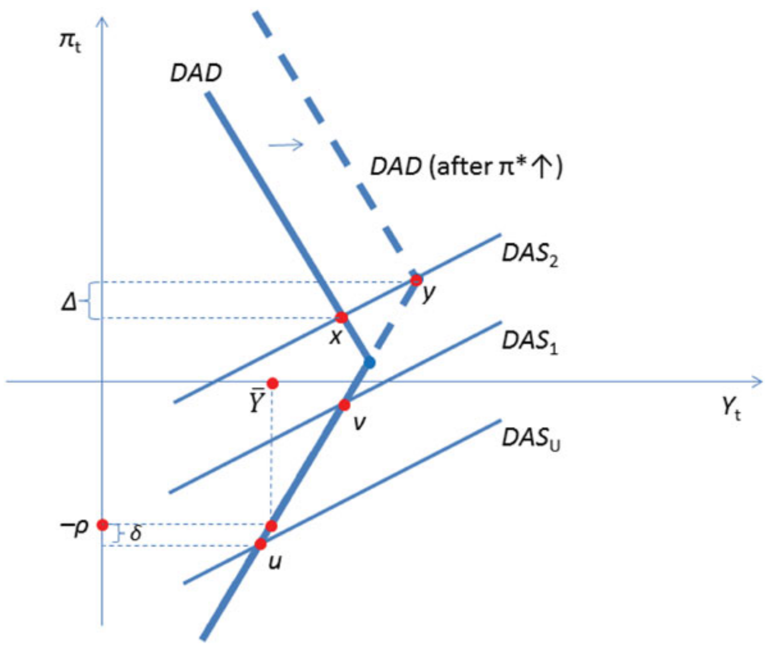

FIGURE 9 Expansionary monetary policy and the deflationary spiral. Both shocks are assumed zero. $\bar{Y}_t = \bar{Y}$ for all t (color figure available online).

Similarly, if the ZLB is binding (say, the economy is at v in figure 9 on the positively sloped part of the DAD curve), monetary policy would not be able to counteract an imminent decline in the DAS curve to DAS_U that would initate a deflationary spiral.

However, suppose the economy is at x in figure 9, and an imminent decrease in the suply shock is expected to take the economy to u. This threatens to reduce the inflation rate to $-\rho - \delta$ and thereby initiate a deflationary spiral. In this case, because the ZLB is nonbinding at x, expansionary monetary policy ($\pi^*\uparrow$) can raise the equilibrium inflation rate by Δ, as a comparison of points x and y shows. And if $\Delta > \delta$, this would be enough to keep the equilibrium inflation rate above $-\rho$ and thereby prevent a deflationary spiral.

To sum up, in the DAD-DAS model, expansionary fiscal policy can be used to rescue an economy that is already in a deflationary spiral and, preemptively, to stop an imminent deflationary spiral. Expansionary monetary policy cannot help if a deflationary spiral is already underway. It may help to keep an economy out of a deflationary spiral, but only if the ZLB on the nominal interest rate has not become binding.

THE ZLB IN PROMINENT TEXTBOOKS

We have shown how the graphical DAD-DAS model in Mankiw (2013) can be easily modified to include the ZLB on the nominal interest rate. We have also shown that the ZLB gives us a deflationary long-run equilibrium and a deflationary spiral. In this section, we will discuss the treatment of these issues in five other intermediate macroeconomics textbooks: Blanchard and Johnson (2013), Carlin and Soskice (2006), Gordon (2012), Jones (2011), and Mishkin (2011).

These five textbooks all discuss the ZLB, and they all make the point that expansionary fiscal policy works at the ZLB whereas expansionary monetary policy (at least of the conventional kind) does not. The five equations of Mankiw's DAD-DAS model—equations (1) through (5)—are present in all five textbooks. However, these five equations are scattered across multiple chapters and are not analyzed together—either graphically or algebraically—as a unified model.

Using monetary policy rules that are somewhat different from Mankiw's rule (5), Carlin and Soskice (2006), Jones (2011), and Mishkin (2011) present graphical models, which, like Mankiw, determine both output and inflation at the intersection of a negatively sloped demand curve and a positively sloped supply curve. Also, like Mankiw, they do not discuss how the ZLB affects their graphs. When we add the ZLB to the models in Carlin and Soskice, Jones, and Mishkin, we again get kinked demand curves. For Carlin and Soskice and Jones, this kinked demand curve has a positively sloped segment for inflation rates below a critical level, as in our modification of Mankiw. For Mishkin, we again get a kink, but with a *vertical* segment instead of a positively sloped segment.

Why the difference? In both Carlin and Soskice (2006) and Jones (2011), the Fisher equation and adaptive expectations yield the usual result that the current real interest rate equals the current nominal interest rate less the current inflation rate: $r_t = i_t - \pi_t$. Therefore, at the ZLB, $r_t = -\pi_t$. As a result, lower current inflation leads to a higher current real interest rate, which, by the IS curve, leads to lower current output, thus yielding a positively sloped aggregate demand curve in (Y, π) space. In Mishkin (2011), however, the Fisher equation is expressed as $r = i - \pi^e$, and adaptive expectation is expressed as $\pi^e = \pi_{-1}$, which is inherited inflation. Therefore, the current real interest rate ($r = i - \pi_{-1}$) is unaffected by current inflation (π). Lower current inflation has no effect on current real interest rate and, therefore, no effect on current output, thus yielding a vertical demand curve below the kink.

It seems natural to us to think that when the nominal interest rate is stuck at zero, lower inflation will lead to higher real interest rates and, therefore, to lower output. However, this persuasive "story" of an economy at the ZLB does not follow from Mishkin's treatment because of the seemingly minor difference in his treatment of the Fisher equation.

Although none of the five textbooks describe our deflationary long-run equilibrium, the textbooks by Carlin and Soskice (2006) and Jones (2011) are distinctive because they provide somewhat informal but intuitive accounts of the deflationary spiral. They explain the deflationary spiral as follows: Suppose ρ is the real interest rate consistent with full employment. If $\pi < -\rho$, then $i = r + \pi \geq 0$ implies $r \geq -\pi > \rho$. Therefore, as the actual real interest rate exceeds the real interest rate consistent with full employment, full employment would not be possible. The resulting recession would drive inflation farther below $-\rho$, and so on and on, causing a deflationary spiral. While this explanation is intuitive, it is not complete (in our view) because current inflation is an *endogenous* variable, and it is simultaneously determined along with current output, the current real interest rate, and the current nominal interest rate. It is necessary to explain why $\pi < -\rho$ would occur in the first place.

Our analysis shows that if *inherited* inflation, which is a predetermined variable, reaches $\pi_{t-1} < \rho$, then a deflationary spiral occurs. To repeat, it is necessary to express the conditions that lead to a deflationary spiral entirely in terms of the model's exogenous givens.

Both Blanchard and Johnson (2013) and Gordon (2012) present a graphical model that determines current output and the current *price level* at the intersection of a negatively sloped demand curve and a positively sloped supply curve. Unlike the other textbooks, Blanchard and Johnson (2013, 199, Fig. 9–10) also show how their demand curve looks under the ZLB: It is kinked, but with a vertical, rather than positively sloped, segment for current inflation rates below a critical level at which the ZLB becomes binding. Blanchard and Johnson (2013, 296) also provide an informal but valuable explanation of the deflationary spiral through an examination of the U.S. economy during the Great Depression.

In discussing the ZLB, Gordon (2012, 250) wrote the following:

> In fact a falling price level increases the *real* interest rate (which is defined as the nominal interest rate minus the rate of inflation; when inflation is negative the real interest rate is higher than the nominal interest rate). A rising real interest rate caused by a falling price level reduces the demand for interest-sensitive consumer durable goods and business investment in equipment and structures and puts further downward pressure on real GDP.

However, a falling price level does not imply falling inflation.

To summarize, although all six textbooks considered here take note of the ZLB, none describes our deflationary long-run equilibrium, and none describes the conditions (in terms of the exogenous variables and parameters of the model economy) under which a deflationary spiral occurs. None of these textbooks describe how the ZLB changes the graphical determination of output and inflation. We have tried to argue that a simple modification of the demand curve addresses all these issues.

CONCLUSION

Several of today's leading textbooks for intermediate macroeconomics courses include a dynamic New Keynesian model of short-run macroeconomics consisting of an IS curve, a Phillips curve, and a monetary policy rule. We have shown in this article that when the DAD-DAS model in Mankiw (2013) is generalized to incorporate the ZLB on the nominal interest rate, it has two long-run equilibria, one stable and the other unstable. We have demonstrated the existence of a deflationary spiral in which both output and inflation fall without bound. We have also described policy responses that can keep an economy out of the deflationary spiral and/or rescue it from such a spiral in case one has already begun.

We realize that a deflationary spiral in which output falls without bound is unrealistic. In Blanchard and Johnson (2013, 178, "Deflation and the Philips Curve Relation"), the authors point out that during the Great Depression, inflation was systematically higher in the United States than predicted by the estimated (or fitted) Phillips curve. Based on this observation, they argue persuasively that workers are reluctant to accept decreases in their nominal wages and that the Phillips curve relation breaks down at low levels of inflation. Adapting this article's model to deal comprehensively with the deflationary spiral remains a topic for future research.

For the time being, note that monetary policy in the United States and Japan (to take just two examples) has been stuck at the ZLB since 2008 and 1995, respectively. Students must see how short-run macroeconomics works under these no longer new—and no longer unusual—circumstances.

NOTES

1. For the graphical analysis in the rest of the article, we will make the simplifying assumption $\bar{Y}_t = \bar{Y}$ for all t.
2. While the algebra of these dynamics are worked out in the appendix, in the body of the article we present a graphical treatment similar in style to that in a typical intermediate macroeconomics textbook.
3. Note from (6) that expansionary monetary policy ($\pi^*\uparrow$) moves the ZLB border upward and to the right—thereby expanding the region where the ZLB is binding—whereas expansionary fiscal policy ($\varepsilon t\uparrow$) has no effect on the ZLB border.
4. The negative feedback loop between output and inflation is the mechanism that leads to a deflation-induced depression, as previously explained by Fisher (1933) and Krugman (1998). In normal times, when nominal interest rates are positive, the central bank can afford to cut interest rates following a negative demand shock to provide short-run stimulus to the economy. When the ZLB is binding, however, cutting rates is not feasible and real interest rates spike up as a result of lower inflation. Higher real rates in turn depress the economy further, which put further pressure on real rates, which depress the economy further, and so on and so forth. As discussed in our section on textbooks, this idea is also explored in several textbooks, such as Gordon (2012).
5. Algebraic proofs of the stability results of this section are given in the appendix.
6. Note that the good news of a favorable cost shock ($\nu_t\downarrow$) (e.g., a fall in the price of imported oil) can trigger a deflationary spiral by lowering the DAS curve, say, from DAS_D to DAS_U. This point has been underscored by Carlstrom and Pescatori (2009): "[T]o be effective in an environment of zero short-term nominal interest rates, monetary policy needs to be unequivocally committed to avoiding expectations of deflation. . . . While this policy prescription follows from the assumption that the zero interest rate bound is a consequence of a negative demand shock hitting the economy, it is worth stressing that falling prices can also be the consequence of a supply shock, namely particularly high productivity growth (not a bad thing!)."
7. See the discussion in our section on stability.

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APPENDIX

AN ALGEBRAIC REPRESENTATION OF THE MODEL

Mankiw (2013, ch. 15) presents numerical simulations of the dynamic adjustment of the DAD-DAS model's economy to various shocks and policy changes. These simulations require that the equilibrium values of the model's endogenous variables be expressed in terms of the model's parameters, shocks, and the predetermined value of inherited inflation. In this appendix, we complete this algebraic task.

When the ZLB is Not Binding

In this section, we assume that the ZLB is not binding. Later in this section, we will specify the conditions (in terms of the model's parameters, shocks, and the predetermined value of inherited inflation) under which the ZLB is not binding.

We have seen the derivation of the DAD curve (7) and the DAS curve (9). The former yields

$$Y_t - \bar{Y}_t = -\frac{\alpha\theta_\pi}{1+\alpha\theta_Y}\left(\pi_t - \pi^*\right) + \frac{1}{1+\alpha\theta_Y}\varepsilon_t.$$

By substituting this for $Y_{\mathrm{t}} - \bar{Y}_t$ in (9), rearranging and collecting the terms, we get the short-run equilibrium inflation:

$$\pi_t = \frac{(1+\alpha\theta_Y)(\pi_{t-1}+\nu_t) + \phi\cdot(\alpha\theta_\pi\pi^* + \varepsilon_t)}{1+\alpha\theta_Y+\alpha\theta_\pi\phi}. \tag{10}$$

Note that equation (10) is simulation-ready. By substituting numerical values for the model's parameters, shocks, and the predetermined value of inherited inflation, we can calculate the numerical value of the current period's inflation. As this period's current inflation is next period's inherited inflation, the exercise can be repeated ad infinitum. Note also that current inflation is increasing in inherited inflation, the cost shock, and the demand shock, as one would expect.

By substituting (10) into the DAD curve (7), we get the short-run equilibrium output:

$$Y_t = \bar{Y}_t + \frac{\alpha\theta_\pi \cdot (\pi^* - \pi_{t-1} - \nu_t) + \varepsilon_t}{1 + \alpha \cdot (\theta_\pi \phi + \theta_Y)}. \tag{11}$$

Note that output increases under expansionary monetary policy ($\pi^*\uparrow$) and/or expansionary fiscal policy ($\varepsilon_t\uparrow$).

By substituting (11) into Mankiw's simplified monetary policy rule [$i_t = \pi_t + \rho + \theta_\pi\cdot(\pi_t - \pi^*) + \theta_Y\cdot(Y_t - \bar{Y}_t)$], we get the short-run equilibrium nominal interest rate:

$$i_t = \rho + \frac{(1 + \theta_\pi + \alpha\theta_Y)(\pi_{t-1} + \nu_t) - (1 - \alpha\phi)\,\theta_\pi\pi^* + (\theta_Y + \phi \cdot (1 + \theta_\pi))\,\varepsilon_t}{1 + \alpha \cdot (\theta_Y + \phi\theta_\pi)}. \tag{12}$$

Equations (2) and (4) together imply that the real interest rate is $r_t = i_t - E_t\pi_{t+1} = i_t - \pi_t$. By substituting equations (10) and (12) into $r_t = i_t - \pi_t$, we get

$$r_t = \rho + \frac{\theta_\pi \cdot (\pi_{t-1} + \nu_t - \pi^*) + (\theta_Y + \theta_\pi\phi)\,\varepsilon_t}{1 + \alpha\theta_Y + \alpha\theta_\pi\phi}. \tag{13}$$

Equation (12) can now be used to derive the conditions under which the ZLB is binding or not. Let π^c_{t-1} be that rate of inherited inflation (π_{t-1}) for which $i_t = 0$ in equation (12). By equating the righthand side of equation (12) to zero and rearranging the terms, we get the following:

$$\pi^c_{t-1} \equiv \frac{(1 - \alpha\phi)\,\theta_\pi\pi^* - (1 + \alpha\theta_Y + \alpha\phi\theta_\pi)\,\rho - (\theta_Y + \phi + \phi\theta_\pi)\,\varepsilon_t}{1 + \alpha\theta_Y + \theta_\pi} - \nu_t. \tag{14}$$

We already know by definition that (a) $i_t = 0$ when $\pi_{t-1} = \pi^c_{t-1}$. As equation (12) implies that the nominal interest rate is strictly increasing in the inherited inflation rate $\partial i_t/\partial\pi_{t-1} > 0$, it follows further that (b) $i_t > 0$ when $\pi_{t-1} > \pi^c_{t-1}$, and (c) $i_t < 0$ when $\pi_{t-1} < \pi^c_{t-1}$. However, we know from (5) that the nominal interest rate cannot be negative. Therefore, we conclude that *the ZLB is binding if and only if* $\pi_{t-1} < \pi^c_{t-1}$. Therefore, the expressions for inflation, output, and the interest rates derived above are valid only when $\pi_{t-1} \geq \pi^c_{t-1}$.

Returning to the equilibrium inflation rate (10) above, it can be checked that if $\varepsilon_t = \nu_t = 0$ and $\pi_{t-1} = \pi^*$ are substituted in equation (10), we get $\pi_{t-1} = \pi_t = \pi^*$. In other words, when there are no shocks, if the inherited inflation is equal to the central bank's target inflation, then the inflation rate repeats itself ad infinitum. This is the orthodox long-run equilibrium of our section on long-run equilibria.

By substituting $\varepsilon_t = \nu_t = 0$ and $\pi_{t-1} = \pi_t = \pi^*$ into equations (11), (12), and (13) above, it is straightforward to show that in the orthodox long-run equilibrium, output is $\bar{Y}_t$, the nominal interest rate is $\rho + \pi^*$, and the real interest rate is ρ.

The stability of the orthodox long-run equilibrium can now be proved. If we subtract π^* from both sides of equation (10) and rearrange and collect the terms, we get, when there are no shocks ($\varepsilon_t = \nu_t = 0$),

$$\pi_t - \pi^* = \frac{1 + \alpha\theta_Y}{1 + \alpha\theta_Y + \alpha\theta_\pi\phi}\left(\pi_{t-1} - \pi^*\right).$$

As $0 < (1 + \alpha\theta_Y)/(1 + \alpha\theta_Y + \alpha\theta_\pi\,\phi) < 1$, it follows that the gap between inflation and the central bank's target inflation retains its sign and shrinks over time (as long as there are no shocks and the model's parameters stay constant). In other words, *if the ZLB is nonbinding* ($\pi_{t-1} \geq \pi^c_{t-1}$)*, the inflation rate* (π_t) *converges monotonically to the orthodox long-run equilibrium inflation rate* (π^*).

It is then straightforward, using equations (11), (12), and (13), that output and the interest rates also converge to their respective orthodox long-run values.

When the ZLB is Binding

We now assume that the ZLB is binding ($\pi_{t-1} < \pi^c_{t-1}$). In this case, the nominal interest rate set by the central bank is $i_t = 0$. Easy!

We have seen the derivation of the DAD curve (8) and the DAS curve (9). The former yields $Y_t - \bar{Y}_t = \alpha \cdot (\pi_t + \rho) + \varepsilon_t$. By substituting this expression for $Y_t - \bar{Y}_t$ into (9), rearranging and collecting the terms, we get the short-run equilibrium inflation rate:

$$\pi_t = \frac{\pi_{t-1} + \nu_t + \alpha\phi\rho + \phi\varepsilon_t}{1 - \alpha\phi}. \tag{15}$$

Simulation-ready expressions for the real interest rate and output can be derived by substituting equation (15) into $r_t = i_t - E_t\pi_{t+1} = i_t - \pi_t = 0 - \pi_t = -\pi_t$ and (8).

It can be checked that if $\varepsilon_t = \nu_t = 0$ and $\pi_{t-1} = -\rho$ are substituted in equation (15), we get $\pi_{t-1} = \pi_t = -\rho$. In other words, when there are no shocks, if the inherited inflation happens to be equal to the negative of the natural (long-run) real interest rate, then that inflation rate repeats itself ad infinitum. This is the deflationary long-run equilibrium of our section on long-run equilibria.

The unstable nature of the deflationary long-run equilibrium can now be proved. If we subtract $-\rho$ from both sides of equation (15) and rearrange and collect the terms, we get the following:

$$\pi_t - (-\rho) = \frac{\pi_{t-1} - (-\rho) + \nu_t + \phi\varepsilon_t}{1 - \alpha\phi}.$$

When there are no shocks ($\varepsilon_t = \nu_t = 0$), this becomes

$$\pi_t - (-\rho) = \frac{1}{1 - \alpha\phi} \cdot (\pi_{t-1} - (-\rho)).$$

As we have assumed $0 < 1 - \alpha\,\phi < 1$ (see our section on the slopes of the DAD and DAS curves), it follows that $1/(1 - \alpha\,\phi) > 1$. Therefore, the gap between current equilibrium inflation and inflation in the deflationary long-run equilibrium retains its sign and increases—in absolute value—over time (as long as there are no shocks and the model's parameters stay constant). In other words, *if the ZLB is binding* ($\pi_{t-1} < \pi^c_{t-1}$)*, the inflation rate* (π_t) *diverges monotonically from the deflationary long-run equilibrium inflation rate* ($-\rho$).

We can summarize our convergence results as follows:

Proposition 1: *Assume there are no shocks. If* $\pi_{t-1} > -\rho$*, the economy converges to the orthodox long-run equilibrium. If* $\pi_{t-1} = -\rho$*, the economy stays in the deflationary long-run equilibrium. If* $\pi_{t-1} < -\rho$*, the economy stays in a deflationary spiral.*