Fiscal policy, long-run growth, and welfare in a stock-flow model of public goods

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Abstract. We introduce public capital and public services as inputs in an endogenous growth model. We show that the growth rate depends on the apportionment of tax revenues between the accumulation of public capital and the provision of public services. When public spending is financed by proportional income taxes, the growth rate, the level of public spending as a proportion of GDP, the level of investment in public capital as a proportion of total public spending, and the level of private investment as a proportion of total private spending all are lower in the equilibrium outcome than in the optimal outcome. JEL classification: E62, O40

Politique fiscale, croissance à long terme et bien-être dans un modèle stock/flow de biens publics. Les auteurs introduisent du capital public et des services publics comme intrants dans un modèle de croissance endogène. On montre que le taux de croissance dépend de l'allocation des revenus de la fiscalité entre l'accumulation de capital public et la fourniture de services publics. Quand les dépenses publiques sont financées par un impôt proportionnel sur le revenu, le taux de croissance, le niveau de dépenses publiques en pourcentage du PIB, le niveau d'investissement en capital public, et le niveau d'investissement privé en proportion de la dépense totale privée sont tous moins élevés au niveau d'équilibre que ce qui serait le résultat optimal

1. Introduction

Since the publication of Romer (1986), the focus in growth theory has been on models that explain long-run growth without appealing to exogenous changes in technology and demographic factors. This is done by introducing

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non-decreasing returns to accumulatable factors as, for instance, in Romer (1989) and Rebelo (1991). Within the framework of growth models with constant returns to a 'broad concept' of capital, Barro (1990) showed how the presence of a *flow* of public *services* as an input in the production function of the final good can affect long-run growth and welfare. Futagami, Morita, and Shibata (1993) and Turnovsky (1997) then introduced a *stock* of public *capital* as an input along the lines of the early work of Arrow and Kurz (1970).[1] Our paper combines these two aspects of productive public spending through a production function that includes public capital and public services. Public capital could be interpreted as being of the nature of roads, railways, airports and other forms of infrastructure that are generally non-rival and non-excludable. Public services include the maintenance of such infrastructure networks as well as other services such as the maintenance of law and order.[2]

Governments routinely have to face trade-offs between the long-term goal of the accumulation of public capital goods and the short-term need to provide public services. Government spending on the formation of public capital, such as spending on research into diseases such as cancer or on construction projects such as the construction of the Suez or Panama canals, typically pays off with a lag. Government spending on public services, such as spending on policing or road maintenance, has a more immediate effect. Therefore, governments, like private individuals, have to choose at every instant between the present and the future. It would be interesting, therefore, to study how changes in the shares of the two components of public spending are related to growth and social welfare and to characterize the optimal mix.

With these goals in mind, we develop in section 2 a model where output is produced with public capital, public services, private capital, and labour. We show that when public spending in the decentralized economy is financed by proportional income taxes, the growth rate of GDP, the level of public spending as a proportion of GDP, the level of investment in public capital as a proportion of total public spending, and the level of investment in private capital as a proportion of total spending on private goods, all are lower in the equilibrium outcome than in the optimal outcome. The ratio in which public capital and public services are used in the production of the final good is, on the other hand, higher in the decentralized equilibrium. The government uses the ratio of the two kinds of public spending as a policy tool to partially offset the non-optimal choices of the private sector. In section 3 we highlight the

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1 However, unlike Futagami, Morita, and Shibata (1993) and Turnovsky (1997), Arrow and Kurz (1970) considered diminishing returns to scale in private and public capital. See also Dasgupta (1999).
2 On the empirical side, the important study by Aschauer (1989) finds that investment in infrastructure does raise the productivity of private capital, leading to higher growth. Easterly and Rebelo (1993) support Aschauer in showing that public investment in transport and communication has a positive impact on growth. See Gramlich (1994) for a detailed survey of the empirical literature in this area.

value added of our model in relation to Barro (1990) and Futagami, Morita, and Shibata (1993) by deriving the main results of these two models as special cases of our (more general) model and by showing that the behaviour of some of the key variables of our model – such as the equilibrium growth rate and tax rate – cannot be interpolated from the behaviour of the corresponding variables in Barro (1990) and in Futagami, Morita, and Shibata (1993). Section 4 concludes the paper.

2. A model of growth with public and private capital accumulation

We begin this section by characterizing the decentralized equilibrium outcome under optimal fiscal policies. We then describe the social optimum outcome and compare the equilibrium and optimum outcomes.

2.1. Equilibrium in a decentralized economy
Consider an economy that has one final good, Y, and let

$$Y_t = F(g_{st}, g_{ft}, K_t, L_t) \tag{1}$$

be the amount of Y that can be produced at time t with g_{st} units of an accumulatable public good (hereafter, *public capital*, a stock variable), g_{ft} units of a non-accumulatable (or, perishable) public good (hereafter, *public services*, a flow variable), K_t units of an accumulatable private good (hereafter, private capital, a stock variable), and L_t units of homogeneous labour. We assume $L_t = 1$ for all t. Consequently, all quantities represent both total and per capita magnitudes. For clarity, we will denote the per capita quantities of the final good and the private capital good at time t by y_t and k_t, respectively.

Public capital and public services are both made out of the final good using a one-for-one technology. Thus, the government's budget constraint is

$$\dot{g}_{st} + g_{ft} = \tau y_t, \tag{2}$$

where τ is the constant income tax rate.[3] The representative consumer's constraint is

$$\dot{k}_t = (1 - \tau) \cdot y_t - c_t, \tag{3}$$

where c_t is (both total and per capita) consumption. The nation's utility is

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt, \tag{4}$$

where $\rho > 0$ is the rate of time preference.

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3 We assume a continuously balanced budget as in Barro (1990) and Futagami, Morita, and Shibata (1993). In Turnovsky (1997), this assumption is relaxed, and the fiscal instruments include public debt.

The *representative consumer's problem* is to choose c_t and $\dot{k}_t$ to maximize utility – which is U in (4) – subject to (3), taking τ, g_{ft}, g_{st} and k_0 as given. The first-order conditions give rise to the Euler equation:

$$\rho + \frac{\dot{c}_t}{c_t} = (1 - \tau) \cdot \frac{\partial y_t}{\partial k_t}. \tag{5}$$

Equation (5) embodies the Keynes-Ramsey rule that the representative consumer's after-tax return from private investment is such that it is impossible for her to increase her lifetime utility by adjusting her rate of private investment. This equation appears in one form or another in all models that have utility-maximizing consumers who choose between consumption and investment and whose income is taxed at a flat rate; see, for example, equation (13) in Barro (1990, S108), equation (5) in Futagami, Morita, and Shibata (1993, 611), and equation (5a) in Bruce and Turnovsky (1999, 166).

The task of the government in a decentralized economy is to run the public sector in the nation's interest, taking the private sector's choices as given.[4] In other words, the *government's problem* is to choose τ, g_{ft} and $\dot{g}_{st}$ to maximize the representative consumer's utility subject to (2), (3), and (5), taking K_0 and g_{s0} as given. The first-order conditions yield

$$\frac{\partial y_t}{\partial g_{ft}} = 1, \tag{6}$$

and

$$\rho + \frac{\dot{c}_t}{c_t} = \frac{\partial y_t}{\partial g_{st}}, \tag{7}$$

which, along with equation (5), yields

$$\rho + \frac{\dot{c}_t}{c_t} = \frac{\partial y_t}{\partial g_{st}} = (1 - \tau) \cdot \frac{\partial y_t}{\partial k_t}. \tag{8}$$

Equation (6) appears in Barro (1990), where it is called the 'natural condition for productive efficiency' (S109). (See also Barro and Sala-i-Martin 1995, 155.) Our assumption that public services are produced out of the final good using a one-for-one technology means that the marginal cost of one unit of public services is one unit of the final good. The marginal benefit, on the other hand, is $\partial y / \partial g_f$. Equation (6) implies that the government's optimal fiscal policy equates the marginal benefits and costs of public services.

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4 This is sometimes called in the literature the government's 'Ramsey policy problem.' See, for instance, Bruce and Turnovsky (1999, 174). See also Sarte and Soares (2003, 41).

Equation (7) has the same Keynes-Ramsey interpretation as equation (5): the return to public capital, $\partial y/\partial g_s$, should be such that it is impossible to increase the representative consumer's lifetime utility by adjusting the rate of investment in public capital. (See equation (25) of Sarte and Soares 2003, 43, for this condition in a discrete-time model that has public capital but not public services.)

For specificity, we assume that the technology available to the firms in the final good sector, equation (1), obeys the Cobb-Douglas form:

$$Y_t = (g_{st}^\alpha \cdot g_{ft}^{1-\alpha})^{1-\beta} \cdot K_t^\beta \cdot L_t^{1-\beta}, \tag{9}$$

where $0 < \alpha$, $\beta < 1$. In per capita terms, this technology can be written as

$$y_t = (g_{st}^\alpha \cdot g_{ft}^{1-\alpha})^{1-\beta} \cdot k_t^\beta. \tag{10}$$

Let $x_t \equiv k_t/(g_{st}^\alpha \cdot g_{ft}^{1-\alpha})$ and $g_{sft} \equiv g_{st}/g_{ft}$. Then equation (6) and the second equality in equation (8) can be expanded, respectively, to

$$(1-\alpha)(1-\beta)x_t^\beta g_{sft}^\alpha = 1 \tag{11}$$

$$\alpha \cdot (1-\beta) \cdot x_t^\beta g_{sft}^{\alpha-1} = (1-\tau) \cdot \beta \cdot x_t^{\beta-1}. \tag{12}$$

One can see that x_t and g_{sft} must be constant over time because (11) and (12) are two equations in which the only dated variables are x_t and g_{sft}.

As both $x_t = k_t/(g_{st}^\alpha \cdot g_{ft}^{1-\alpha})$ and $g_{sft} = g_{st}/g_{ft}$ are constant along the balanced growth equilibrium path, k_t, g_{st}, and g_{ft} all must grow at the same rate γ_t.[5] And, by equation (10), γ_t will then also be the growth rate of y_t. That is,

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{k}_t}{k_t} = \frac{\dot{g}_{st}}{g_{st}} = \frac{\dot{g}_{ft}}{g_{ft}} = \gamma_t. \tag{13}$$

Equation (2), which implies $\dot{g}_{st}/g_{st} = \tau \cdot (y_t/g_{st}) - (g_{ft}/g_{st})$, and (13) together yield

$$\gamma_t = \tau \cdot x_t^\beta \cdot g_{sft}^{\alpha-1} - g_{sft}^{-1}, \tag{14}$$

which is constant over time, since τ, x_t and g_{sft} are all constant over time. So, let $\gamma_t = \gamma$.

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5 Let the ratio k_t/g_{st} be denoted by m_t. For the equilibrium outcome, note that equation (12) implies that, for the Cobb-Douglas case, $m_t \equiv k_t/g_{st} = x_t \cdot g_{sft}^{\alpha-1} = \alpha^{-1} \cdot \beta \cdot (1-\beta)^{-1} \cdot (1-\tau) \equiv m$, a constant. Here, k_t and g_{st} are state variables and, therefore, k_0 and g_{s0} are given data. Consequently, $m_0 = k_0/g_{s0}$ will not, except by coincidence, be equal to the expression given above. Therefore, given the constraints $k_t \geq 0$ and $\dot{g}_{st} \geq 0$, we need to work out the equilibrium outcome separately for all arbitrary values of $m_0 = k_0/g_{s0}$. This can be done along the lines of section 5.1.2 of Barro and Sala-i-Martin (1995). The details of the transitional dynamics for the equilibrium outcome are provided in an appendix available upon request.

Similarly, equation (3), which implies $\dot{k}_t/k_t = (1-\tau)\cdot(y_t/k_t) - (c_t/k_t)$, and (13), which implies that y_t/k_t is constant, and the previous paragraph's result that $\dot{k}_t/k_t = \gamma_t = \gamma$ together imply that c_t/k_t must be constant along the balanced growth path. Therefore, we have

$$\gamma_t = \gamma = \frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \frac{\dot{g}_{st}}{g_{st}} = \frac{\dot{g}_{ft}}{g_{ft}}. \tag{15}$$

Now, the two equalities in (8) give us, in order,

$$\gamma_t = \frac{\partial y_t}{\partial g_{st}} - \rho = \alpha \cdot (1-\beta) \cdot x_t^\beta \cdot g_{sft}^{\alpha-1} - \rho \tag{16}$$

$$\gamma_t = (1-\tau) \cdot \frac{\partial y_t}{\partial k_t} - \rho = (1-\tau) \cdot \beta \cdot x_t^{\beta-1} - \rho. \tag{17}$$

Equations (11), (14), (16), and (17) are four independent equations in four unknowns: x_t, g_{sft}, τ, and γ_t. Therefore, the equilibrium values of these variables are determined in terms of α, β, and ρ and all must, therefore, be constant over time as we have already seen. Unfortunately, closed-form solutions are not obtainable, owing to the non-linearities in these equations. Nevertheless, it can be checked that the decentralized economy's *equilibrium outcome* – denoted by the superscript '*e*' – under the optimal management of the public sector must satisfy the following equations:

$$g_{sft} = g_{sf}^e = [\alpha^\beta \cdot (1-\alpha)^{-1} \cdot \beta^{-\beta} \cdot (1-\beta)^{\beta-1} \cdot (1-\tau^e)^{-\beta}]^{\frac{1}{\alpha-\alpha\beta+\beta}} \tag{18}$$

$$x_t = x^e = [(1-\alpha)^{-1} \cdot (1-\beta)^{-1} \cdot (g_{sf}^e)^{-\alpha}]^{1/\beta} \tag{19}$$

$$\gamma_t = \gamma^e = \alpha \cdot (1-\alpha)^{-1} \cdot (g_{sf}^e)^{-1} - \rho \tag{20}$$

$$\tau = \tau^e = (1-\beta) \cdot [1 - (1-\alpha) \cdot \rho \cdot g_{sf}^e]. \tag{21}$$

These expressions will allow us to compare the equilibrium outcome with the optimal outcome below. It will be particularly useful to note that equations (19)–(21) follow directly from equations (11), (14), and (16).[6,7]

Next, let us look at how the government allocates the tax revenues between $\dot{g}_{st}$, its accumulation of public capital, and g_{ft}, its provision of public services.

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6 Futagami, Morita, and Shibata (1993) show analytically that the welfare-maximizing tax rate in the decentralized economy is lower than the growth-maximizing tax rate in their model with public capital (620–2), but do not derive an explicit expression for this welfare-maximizing tax rate. In our paper, we can obtain a numerical solution for this tax rate (τ^e) from equation (21) after eliminating g_{sf}^e, using equation (18).

7 We are considering only those values of α, β, and ρ for which τ, θ, and s are positive fractions – in both equilibrium and optimum – in conformity with their definitions. See the sentence that includes equation (24) for a definition of s.

Let the share of the total output of public goods that goes to the accumulation of public capital be

$$\theta_t \equiv \frac{\dot{g}_{st}}{\dot{g}_{st} + g_{ft}}. \tag{22}$$

Equations (22) and (20) imply

$$\theta_t = \theta^e = 1 - \frac{1}{(1-\alpha)^{-1} - \rho \cdot g_{sf}^e}. \tag{23}$$

Finally, s_t, the private saving rate, is given by $s_t = \dot{k}_t/(1-\tau)\cdot y_t = (\dot{k}_t/k_t)/(1-\tau) \cdot (y_t/k_t)$, which simplifies to

$$s_t = s^e = \gamma^e \cdot (1-\tau^e)^{-1} \cdot (x^e)^{1-\beta}. \tag{24}$$

2.2. Optimum: the social planner's problem

To describe the social planner's problem we need to first redefine τ and θ_t. Let τ now represent the share of total output that is devoted by the social planner to the twin tasks of accumulating public capital and providing public services and let θ_t now represent the share of the total expenditure on the two public goods that is devoted by the social planner to the accumulation of public capital.[8] The social planner's objective is to maximize utility – which is U in equation (4) – and equations (2) and (3) constrain the planner. The planner must choose c_t, the level of private consumption, and $\dot{k}_t$, the level of investment in *private* capital. In short, the *social planner's problem* is to choose c_t, g_{ft}, τ, $\dot{g}_{st}$ and $\dot{k}_t$ so as to maximize U subject to equations (1)–(3), $L_t = 1$, and the given amounts, K_0 and g_{s0}, of the two capital goods that the economy starts out with. The first-order conditions give us equation (6), as in our earlier analysis of the decentralized outcome, and

$$\rho + \frac{\dot{c}_t}{c_t} = \frac{\partial y_t}{\partial g_{st}} = \frac{\partial y_t}{\partial k_t}. \tag{25}$$

We have discussed the rationale behind equation (6) in section 2.1 above. Note that equation (6) is equivalent to equation (11).

The Keynes-Ramsey rule in equation (25) requires the planner to ensure that the rates of return to private and public capital are such that the representative consumer's lifetime utility cannot be increased by adjusting the rates of investment in those two kinds of capital.

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8 Recall that in our discussion of the decentralized economy, τ was the tax rate and θ_t was the proportion of tax revenues that was used for the accumulation of public capital. The terms 'tax rate' and 'tax revenues' have no meaning in any discussion of the social planner's problem. This necessitated the new definitions used in this section. These definitions of τ and θ_t in section 2.2 would work well in section 2.1, but not the other way around.

Note the contrast with equation (8), which is the counterpart of equation (25) in the case of the decentralized economy. This contrast between equations (8) and (25) encapsulates the wedge that is driven between the equilibrium and optimum outcomes by the use of the income tax to finance public spending in the decentralized economy.

The second equality in equation (25) can be expanded to

$$\alpha \cdot (1 - \beta) \cdot x_t^{\beta} g_{sft}^{\alpha - 1} = \beta x_t^{\beta - 1}. \tag{26}$$

Since equations (11) and (26) are true along the optimal path, one can see that x_t and g_{sft} must be constant over time because (11) and (26) are two equations in which the only dated variables are x_t and g_{sft}.

The constancy of x_t and g_{sft} along the optimal path and equation (10) imply that equation (13) continues to hold in the optimum. Since (2), (10), and (13) are satisfied along the optimal path, equation (14) must be, too. Therefore, y_t, k_t, g_{st}, and g_{ft} all must grow at the same rate along the optimal path and that rate must be constant, as before.

Equation (3), which implies $\dot{k}_t/k_t = (1 - \tau) \cdot (y_t/k_t) - (c_t/k_t)$ and is satisfied by the optimal path, yields the result that y_t, k_t, g_{st}, g_{ft}, and c_t all must grow at the same constant rate along the optimal path. That is, equation (15) is satisfied along the optimal path. Then, the two equalities in (25) give us, in order, equation (16) and

$$\gamma_t = \frac{\partial y_t}{\partial k_t} - \rho = \beta \cdot x_t^{\beta - 1} - \rho. \tag{27}$$

Note the contrast between equations (27) and (17). This is a replay of the divergence between equations (8) and (25).

We have shown, therefore, that equations (11), (14), (16), and (27) are four independent equations that are satisfied along the optimal path and have four unknowns: x_t, g_{sft}, τ, and γ_t. Therefore, the optimal values of these variables are determined in terms of α, β, and ρ, and all must, therefore, be constant over time, as we saw earlier. Specifically, it can be checked that the *optimum outcome* or the solution to the social planner's problem – denoted by asterisks – is as follows:

$$g_{sft} = g_{sf}^* = [\alpha^{\beta} \cdot (1 - \alpha)^{-1} \cdot \beta^{-\beta} \cdot (1 - \beta)^{\beta - 1}]^{\frac{1}{\alpha - \alpha\beta + \beta}} \tag{28}$$

$$x_t = x^* = [(1 - \alpha)^{-1} \cdot (1 - \beta)^{-1} \cdot (g_{sf}^*)^{-\alpha}]^{1/\beta} \tag{29}$$

$$\gamma_t = \gamma^* = \alpha \cdot (1 - \alpha)^{-1} \cdot (g_{sf}^*)^{-1} - \rho \tag{30}$$

$$\tau = \tau^* = (1 - \beta) \cdot [1 - (1 - \alpha) \cdot \rho \cdot g_{sf}^*]. \tag{31}$$

Note that (28) provides a closed-form solution for g_{sft} in terms of the parameters α, β, and ρ and that (29)–(31), therefore, provide solutions for x_t, γ_t and τ in terms of α, β, and ρ.

Recall that the equilibrium outcome, summarized by equations (18)–(21), was derived from equations (11), (14), (16), and (17), whereas the optimum outcome, summarized by equations (28)–(31), was derived from equations (11), (14), (16), and (27). The seemingly slight difference between equations (17) and (27), the only equations not shared by the two sets of equations, turns out to be enough to make closed-form solutions obtainable in one case and unobtainable in another.

As in section 2.1, θ_t, the share of the total production of public goods that goes to the accumulation of public capital, is given by equations (22) and (30), which imply

$$\theta_t = \theta^* = 1 - \frac{1}{(1-\alpha)^{-1} - \rho \cdot g_{sf}^*}. \tag{32}$$

Finally, s_t, the share of the total output of private goods that is invested in private capital, is given by $s_t = \dot{k}_t/(1-\tau)\cdot y_t = (\dot{k}_t/k_t)/(1-\tau)\cdot(y_t/k_t)$, which simplifies to

$$s_t = s^* = \gamma^* \cdot (1 - \tau^*)^{-1} \cdot (x^*)^{1-\beta}. \tag{33}$$

2.3. Equilibrium and optimum compared

In this section, we compare the equilibrium outcome, as summarized by equations (18)–(24), with the optimum outcome, as summarized by equations (28)–(33).[9]

Since the optimal tax rate in the decentralized economy is neither zero nor 100% but somewhere in between – that is, $0 < \tau^e < 1$; see fn. 7 – it then follows directly from a comparison of (18) and (28) that

$$g_{sf}^e > g_{sf}^* \tag{34}$$

for all α, β, and ρ. This result will be used below to compare the equilibrium and optimal values of the other variables of interest.

By comparing equations (19)–(23) with equations (29)–(32) in the light of equation (34), we get

$$x^e < x^* \tag{35}$$
$$\gamma^e < \gamma^* \tag{36}$$
$$\tau^e < \tau^* \tag{37}$$
$$\theta^e < \theta^*. \tag{38}$$

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9 Note that Turnovsky (1997) in his model with private and public capital (but no public services) derives an expression for optimal government expenditure (624), but for the decentralized economy he treats the public investment to output ratio as being set arbitrarily.

Although our interest in $x = k/(g_s^\alpha g_f^{1-\alpha})$ itself is limited, equation (35) is nonetheless useful because a comparison of (24) and (33) in the light of (35)–(37) implies

$$s^e < s^*. \tag{39}$$

In other words, the share of the total output of private goods that is used for the accumulation of private capital in equilibrium is sub-optimal.

The reasoning behind this result follows from a comparison of the Euler equations (8) and (25), which apply to the equilibrium and optimum outcomes, respectively. When the representative consumer in the decentralized economy considers the sacrifice of a unit of current consumption, she sees the after-tax remnant of the marginal product of (private) capital as the future payoff. Although the part that is taken away by the government in taxes is not wasted but is used to produce the public goods that are essential for production, the representative consumer – reflecting the familiar assumption that she is atomistic – concludes that she would not be able to affect the public sector appreciably through her own choices. The omniscient social planner, on the other hand, takes the entire marginal product as the future payoff and therefore devotes a larger share of the output of private goods to the accumulation of private capital.

This divergence between the incentives of the representative consumer and the social planner is at the root of equations (34)–(38) as well. In particular, equation (36) says that the optimal rate of growth exceeds the equilibrium rate of growth. This is a direct consequence of the slower rate of private capital accumulation in equilibrium – due to the coordination failure among atomistic agents – which we discussed in the previous paragraph.

Equation (37) says that the share of total output that is devoted to the production of the two public goods in equilibrium is sub-optimal. To see why, note that the government in the decentralized economy is aware (i) that public goods are financed by income taxes and (ii) that such taxes generate dead-weight losses. The hypothetical social planner in the social optimum outcome, on the other hand, can, by definition, provide public goods without any dead-weight losses. Therefore, it follows that the government in the decentralized economy will put a lower emphasis on public goods than the social planner.

Equation (35) can be explained in terms of the government's response, in the decentralized economy, to the representative consumer's perception of the future payoff to the current sacrifice. The payoff to the accumulation of private capital as the representative consumer sees it, $(1 - \tau) \cdot \partial y/\partial k$, is lower than $\partial y/\partial k$, which is the payoff as the social planner would see it. The government in the decentralized economy, therefore, does what it can to raise the equilibrium value of $(1 - \tau) \cdot \partial y/\partial k$. One way to do this is to keep τ low in equilibrium, which is why $\tau^e < \tau^*$, as was discussed earlier. Another way – given that $\partial y/\partial k = \beta x^{\beta-1}$ – is for the government to use its indirect control over x to keep x low in equilibrium, which is why $x^e < x^*$.

Making x^e diverge from x^* so as to adjust $\partial y/\partial k$, also requires that g_{sf}^e diverge from g_{sf}^* in order to bring about the parallel adjustment to $\partial y/\partial g_s$ that is required by the second equality in equation (8). This leads to equation (34).

The divergence between equilibrium and optimum values of the policy variables τ and θ does not imply that the government in the decentralized economy is making a mistake; it is simply trying to compensate for the coordination failure arising out of the atomistic agent's perception of the true future reward for current sacrifice. The government in the decentralized economy could easily have set $\tau^e = \tau^*$ and $\theta^e = \theta^*$, as these variables are under the government's total control. The representative consumer's non-optimal choices, however, induces the government also to choose non-optimal values.

The key point is that the simultaneous presence of public capital and public services in this paper's model provides the government in a decentralized economy with an additional policy tool that is not available in models that have either public capital, as in Futagami, Morita, and Shibata (1993) and Turnovsky (1997), or public services, as in Barro (1990), but not both. By adjusting τ and θ appropriately, it is possible to partially correct the representative consumer's non-optimal choices.

Finally, we would like to point out that the non-optimality of the decentralized equilibrium is not caused by the presence of the two public goods in our model. It is straightforward to show – as indeed we have shown in an appendix available on request – that the optimum outcome attained by the social planner can be reproduced in a decentralized economy if public spending is financed by either a consumption tax or a tax on income from (inelastically supplied) labour. This implies that it is the government's reliance on the income tax that is at the root of the non-optimality of the decentralized equilibrium in our model.

3. Comparisons with Barro (1990) and Futagami, Morita, and Shibata (1993)

As we said in our introduction, Barro (1990) presents a growth model in which the government provides public services but does not invest in public capital, whereas Futagami, Morita, and Shibata (1993) present a growth model in which the government invests in public capital but does not provide public services. In our model the government does both of those things. As a result, only in our model does the government face the trade-off between short-term concerns (the provision of public services) and long-term needs (the accumulation of public capital). Aside from the novelty of the introduction of this trade-off, however, one might wonder whether building a model in which the government faces this trade-off changes the way we had been taught to think about the link between growth and public spending by Barro (1990) and Futagami, Morita, and Shibata (1993).

In this section, we demonstrate that it would be a mistake to think that by bringing together the insights of Barro (1990) and Futagami, Morita, and Shibata (1993) one would be able to intuit the results of our general model. We show that important features of the comparative static behaviour of variables such as the growth rate and the share of public spending to total output in our general model cannot be inferred by comparing the behaviour of those variables in the Barro (1990) and Futagami, Morita, and Shibata (1993) models.

Since Barro (1990) is essentially identical to our model except that it does not have public capital, Barro's model has the following characteristics: the budget constraint of the government is $g_{ft} = \tau \cdot y_t$, the budget constraint of the representative agent is $\dot{k}_t = (1 - \tau) \cdot y_t - c_t$, as in our model, and the technology of the final good sector is $y_t = g_{ft}^{1-\beta} \cdot k_t^{\beta}$. If we now reapply the method used in section 2.1 for the derivation of the decentralized equilibrium, we get

$$\gamma^e = \beta^2 \cdot (1 - \beta)^{(1-\beta)/\beta} - \rho \equiv \gamma_B^e \tag{40}$$

$$\tau^e = 1 - \beta \equiv \tau_B^e. \tag{41}$$

It can be checked that, as one would expect, these equations also follow from equations (20) and (21) for the special case of $\alpha = 0$.[10]

Futagami, Morita, and Shibata (1993), on the other hand, is essentially identical to our model, except that it has no public services. Therefore, their model has the following characteristics: the budget constraint of the government is $\dot{g}_{st} = \tau \cdot y_t$, the budget constraint of the representative agent is $\dot{k}_t = (1 - \tau) \cdot y_t - c_t$, as before, and the technology of the final good sector is $y_t = g_{st}^{1-\beta} \cdot k_t^{\beta}$. If we now reapply the method used in section 2.1 for the derivation of the decentralized equilibrium, we get

$$\gamma^e = (1 - \tau^e)^{\beta} . \beta^{\beta} \cdot (1 - \beta)^{(1-\beta)} - \rho. \tag{42}$$

$$\tau^e = (1 - \beta) \cdot [1 - \rho \cdot (1 - \tau^e)^{-\beta} . \beta^{-\beta} . (1 - \beta)^{(\beta-1)}]. \tag{43}$$

These are two equations in two unknowns, γ^e and τ^e. Therefore, γ^e and τ^e can be numerically solved for specified values of β and ρ, and we will refer to these solutions as γ_F^e and τ_F^e. It can also be checked that, quite predictably, these equations follow from equations (20) and (21) for the special case of $\alpha = 1$.[11]

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10 In the Barro model, the consumer's optimization problem and the first-order conditions are exactly the same as in our general model. These give rise to the same Euler equation as our equation (5).

11 Note that, unlike us, Futagami, Morita, and Shibata (1993) do not derive an expression for the equilibrium tax rate (or the optimal tax rate, for that matter). The focus of their paper is to demonstrate that in a model with public capital (unlike Barro 1990), the welfare-maximizing tax rate for the market economy is different from (i.e., lower than) the growth-maximizing tax rate, owing to the existence of transitional dynamics.

TABLE 1
Solutions of equations (18)–(21) for γ^e. Here $\rho = 0.04$

	$\beta = 0.40$	$\beta = 0.60$
$\alpha = 0$, Barro (1990)	0.0343613	0.155438
$\alpha = 0.10$	0.0331451	0.139102
$\alpha = 1$, Futagami, Morita, and Shibata (1993)	0.335238	0.350671

TABLE 2
Solutions of equations (18)–(21) for τ^e. Here $\rho = 0.04$

	$\beta = 0.45$	$\beta = 0.55$
$\alpha = 0$, Barro (1990)	0.55	0.45
$\alpha = 0.60$	0.475603	0.399108
$\alpha = 1$, Futagami, Morita, and Shibata (1993)	0.490689	0.402453

By substituting various values of α, β, and ρ in equations (18), (20), and (21) – which are three equations in three unknowns: g_{sf}^e, γ^e, and τ^e – we can compute the corresponding values of g_{sf}^e, γ^e, and τ^e. In particular, values of γ^e and τ^e have been tabulated in tables 1 and 2. These same values of γ^e and τ^e could also have been obtained from equations (40) and (41) for the $\alpha = 0$ case and from equations (42) and (43) for the $\alpha = 1$ case.[12]

Seeing that γ^e, the balanced growth rate for the decentralized economy under optimal fiscal policies, goes from 0.0344 to 0.3352 as α goes from zero to unity when $\beta = 0.40$ and $\rho = 0.04$, one might be tempted to guess that, in our general model, the value of γ^e for an intermediate value of α – such as $\alpha = 0.10$ – would lie somewhere between 0.0344 and 0.3352. But, as we see in table 1, the value of γ^e is in this case 0.0331, which is lower than both 0.0344 and 0.3352. In other words, there is a U-shaped dependence of γ^e on α. As a result, if we rely only on Barro (1990) and Futagami, Morita, and Shibata (1993) to make an 'educated guess' about how γ^e would behave in our general model, we would be making a mistake. This indicates that our model, by making the government deal with a trade-off between the provision of public services and the accumulation of public capital, delivers insights that are not available from models that have either the former or the latter but not both.

Table 1 shows a similar U-shaped dependence of γ^e on α for $\beta = 0.60$ and $\rho = 0.04$. Table 2 shows that τ^e, the optimal income tax rate for the decentralized economy, also has a U-shaped dependence on α when $\rho = 0.04$ and β is

12 The full set of simulation results on how the equilibrium values of the key variables of our general model, namely, those given by equations (18)–(21), (23), and (24), change with α, are obtainable upon request.
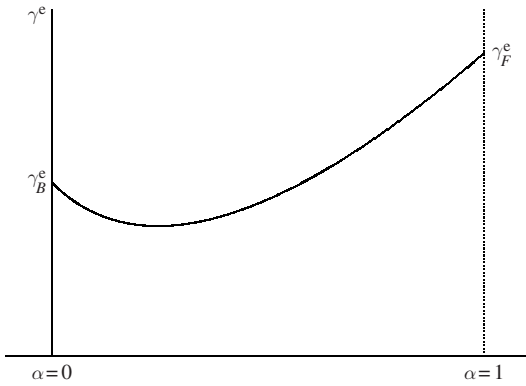
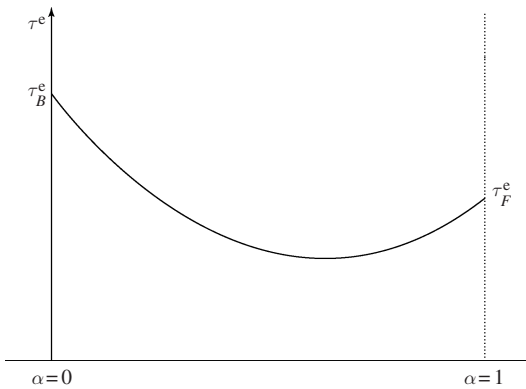FIGURE 1   Effect of α on γ^e in our model

FIGURE 2   Effect of α on τ^e in our model

either 0.45 or 0.55. Therefore, the argument made in the paragraph before last might just as easily have been based on the behaviour of τ^e.

  To repeat, the examples in tables 1 and 2 demonstrate a U-shaped dependence of γ^e and τ^e on α, and this reflects the fact that our generalized model reveals more about the effect of α on γ^e and τ^e than a simple-minded interpolation from the $\alpha = 0$ and $\alpha = 1$ cases would reveal. The results of tables 1 and 2 are summarized with the aid of two diagrams showing the U-shaped dependence of γ^e on α and τ^e on α respectively.

  The same point can also be made by looking instead at the *optimal* values, γ^* and τ^*, obtained from equations (28), (30), and (31). These, too, are U-shaped in α.

4. Conclusion

In this paper we have introduced two public goods in the production function of the final good sector: one that can be accumulated, called public capital, and another that cannot be accumulated, called public services. This allows us to explore the implications of the public sector's present-versus-future trade-off. Our model highlights the idea that the government's effect on an economy depends not only on the tax rate but also on the apportionment of tax revenues between the provision of public services and the accumulation of public capital. The latter policy tool can be used not only to affect the rate of the economy's growth but also to partially bridge the divergence between equilibrium and optimum.

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