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VERTICAL PRODUCT DIFFERENTIATION AND THE VALUE OF TIME

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The paper presents a model of international trade in goods that are ranked by quality. The model differs from existing theories of trade under vertical product differentiation in several ways. The main difference is in the way quality is modeled. But this paper also has different positive implications for the pattern of trade and it provides a theoretical explanation for Leontief Paradox-type empirical findings. The model indicates that several of the most important empirical challenges faced by the Heckscher-Ohlin-Vanek model can be dealt with by taking into account the quality aspect of traded goods. [F1]

1. INTRODUCTION

This paper presents a model of trade in goods that are ranked by quality. The model differs from existing theories of trade under vertical product differentiation, such as Flam and Helpman (1987), Falvey and Kierzkowski (1987), and Stokey (1991), in several ways. The main difference is in the way quality is modeled. But this paper also has different positive implications for the pattern of trade and it provides a theoretical explanation for Leontief Paradox-type empirical findings. The model indicates that several of the most important empirical challenges faced by the Heckscher-Ohlin-Vanek (HOV) model can be dealt with by taking into account the quality aspect of traded goods.

There are essentially two approaches to the modeling of quality in the literature on trade under vertical product differentiation. One approach—adopted by Falvey and Kierzkowski (1987) and Flam and Helpman (1987)—is to have a continuum of goods indexed by q and to postulate a utility function $U(q,x,y)$, where x is the amount consumed of good q and y is the amount consumed of some homogeneous good Y. To ensure that an increase in a consumer's income will lead to the purchase of a superior quality (i.e., to an increase in q) rather than the purchase of greater quantities of the same quality (i.e., an increase in x with no increase in q) Falvey and Kierzkowski make preferences non-homothetic in such a way that the income elasticity of demand becomes increasing in q. Flam and Helpman on the other hand fix $x = 1$, thus ensuring that a consumer whose income has increased will not have the option of buying more of the same good. The second approach is that of Stokey. Here one unit

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of a product denoted $(\underline{q}, \bar{q})$ contains one unit each of the Lancasterian characteristics $\underline{q}$ through $\bar{q}$, where $\underline{q} \equiv 0$ and $\bar{q} > 0$. One good is said to be superior in quality to another if its $\bar{q}$ is bigger. As in Falvey and Kierzkowski—and for the same reasons—preferences are assumed to be non-homothetic.

Notice that in these models the key factor that distinguishes consumers who buy superior goods from consumers who buy inferior goods is income. In my paper, on the other hand, the relevant factor is income *from work*. The model makes use of the assumption, first introduced by Becker (1965), that consumption and the receipt of the pleasures therefrom take time. It is assumed that there is a continuum of goods. Each good is seen as a delivery system for a desirable Lancasterian characteristic that one may, for simplicity, call "pleasure". The quality of a good is defined as the amount of pleasure that the good delivers per unit of time. (It seems proper to define the quality of a good as its contribution to the quality of life of its consumers. Of course, it may be argued that the quality of a life depends not only on how pleasant that life is but also on how long it is and perhaps on still other factors. But to the extent that the various goods in my model are neutral in their effects on longevity and those other factors, it makes sense to equate the quality of a good with the amount of pleasure it delivers per unit of time.) It is then straightforward to show that higher qualities will be consumed by those who earn higher *wages* and, consequently, have a higher opportunity cost of time.

In free trade between North (a country with workers of high ability) and South (a country with workers of low ability), North will consume a high quality good and South will consume a low quality good. Assuming trade takes place, South, therefore, will export a high quality good to North and North will export a low quality good to South. In the models of Flam and Helpman, Falvey and Kierzkowski, and Stokey, on the other hand, South buys high quality goods from North and North buys low quality goods from South. One reason for this difference in trade patterns is that in Flam and Helpman and in Falvey and Kierzkowski the income distributions of North and South overlap and, therefore, there are some rich (resp. poor) people in South (resp. North) who buy Northern (resp. Southern) goods. Were there no overlap in demand patterns, there would be no trade. In the case of Stokey, the model of vertical product differentiation is actually based on consumers' taste for variety or *horizontal* product differentiation. Again, this generates an overlap of Northern and Southern demand patterns. In my model, on the other hand, demand patterns do not overlap; Southerners consume one good and Northerners another. Besides, both Flam and Helpman (1987) and Falvey and Kierzkowski (1987) have an undifferentiated (or homogeneous) good that both countries need and South can produce cheaply. The presence of the homogeneous good, aside from helping to explain the trade pattern, makes the trade in these models both intra-industry and inter-industry. My paper, on the other hand, focuses on the economics of intra-industry trade with vertical product differentiation.

The contrasts between this paper's model and the HOV model are also interesting. First, in this paper's model (and in the other models of vertical product differentiation) there are stark differences in the consumption patterns of North and South; the low-productivity country (South) consumes a low quality good and the high-productivity country (North) consumes a high quality good. In HOV, on the other hand, the assumption of identical and homothetic preferences implies identical consumption patterns in North and South. Markusen (1986) quotes data from Kravis et al. (1982) and argues that there are significant differences in consumption patterns across countries. For instance, 60% of India's personal consumption expenditure is spent on food while for the US it is only 13%.

Secondly, my model attempts to show why the levels of North-South trade are low (see Markusen 1986 for more on this). There is no explanation for this in HOV. In HOV the main source of gains from trade is the international variation in the per capita stock of capital. The great differences in the per capita capital stocks of North and South would, therefore, indicate, contrary to fact, vigorous trade between the two. In my model, on the other hand, high (resp. low) ability North (resp. South) consumes a high (resp. low) quality good that is capital (resp. labor) intensive, and since South (resp. North) is labor (resp. capital) abundant, it does not have a comparative advantage in its production. This reduces the level of North-South trade.

And lastly, this paper's predictions about the connection between factor abundance and the factor content of trade are different from those of HOV. It is assumed in this paper that the goods that are higher in quality are more capital intensive than the goods that are lower in quality. Thus the good that South exports is always more capital intensive than the good that North exports—irrespective of whether South is labor or capital abundant relative to North. In the HOV model, on the other hand, the labor (resp. capital) abundant country always exports the labor (resp. capital) intensive good. The empirical evidence on this issue is mixed. Beginning with Leontief (1953) several studies have found that the HOV trade pattern is often rejected by the data. To take the most recent example, in Trefler's (1995) study of 1983 data for 33 countries the HOV prediction turns out to be true only 49.8% of the time, prompting Trefler to comment: "... the HOV prediction is about as good as a coin toss". The present paper's model shows that the Leontief Paradox-type findings make perfect sense in a model of intra-industry trade under vertical product differentiation.

An earlier paper by Trefler, Trefler (1993), suggests a different approach to this issue; keep the HOV model, but redefine all resources in effective, or productivity, units. Trefler argues that the HOV prediction when expressed in terms of effective units does much better empirically. Note, however, that the modified HOV of Trefler (1993) still does not explain why consumption patterns differ internationally and why North-South trade levels are low. As long as the international variation in demand patterns is not addressed, no modification of the HOV model will be entirely satisfactory.

Section 2 describes the model, section 3 discusses the main results, section 4 discusses the proofs, and section 5 concludes the paper.

2. TRADE AND QUALITY

Let there be two countries, $j = 1,2$. Each of the N_j citizens of country j are endowed with one unit of time. Time is spent on consumption as well as at work. If t_j units of time are spent at home by a citizen of j, then $1 - t_j$ units of time are spent at work. There are a continuum of goods, indexed by $\upsilon > 0$. These consumption goods are assumed to be perfect substitutes in the sense that the consumption of one unit of some good is assumed to be as satisfying as the consumption of one unit of any other good. A worker with ability h_j working $1 - t_j$ units of time supplies $h_j(1 - t_j)$ units of effective, or standard, labor time. Each citizen of j is also endowed with capital; the total capital endowment of country j is K_j. The technology for the production of good υ is the same in both countries; good υ can be produced out of capital K and effective labor time L according to the concave, constant returns to scale technology $F(K,L;\upsilon)$. Labor and capital are immobile internationally, but international trade in final goods is free and all markets are competitive.

Let $N \equiv (N_1, N_2)$, $K \equiv (K_1, K_2)$, and $h \equiv (h_1, h_2)$, and let $z \equiv (N,K,h)$ denote a world consisting of countries 1 and 2. It will be assumed without loss of generality that the unit of account in which prices are expressed is the same in both countries. Moreover, for the purposes of this paper, it will only be necessary to discuss free trade equilibria in which factor prices are equalized as a result of trade in commodities. Therefore, we can let ω_L and ω_K denote factor prices in the two countries. If good υ is produced, its price $p(\upsilon)$ must equal its unit cost, denoted $p(\omega_K, \omega_L,\upsilon)$, because production takes place under perfect competition with concave, constant returns technologies. In this paper, however, $p(\upsilon) = p(\omega_K, \omega_L,\upsilon)$ will be assumed whether or not υ is produced in equilibrium. It will be argued later that this involves no loss of generality.

A. CONSUMER BEHAVIOR

It is assumed that the typical consumer must choose which n goods, $\upsilon^1, \upsilon^2, \ldots, \upsilon^n$, to consume, and what amounts, $c^1, c^2, \ldots, c^n$, of those goods to consume. Formally, the choice problem of a price taking consumer in country j with ability h_j and capital k is to choose υ^i and c^i, $i = 1, 2, \ldots, n$, and t so as to

$$\max U\left(\sum_{i=1}^{n} c^i,\ t\right)$$

$$\text{subject to} \quad \sum_{i=1}^{n} c^i p(\omega_K, \omega_L, \upsilon^i) \le \omega_L h_j (1 - t) + \omega_K k \tag{1}$$

$$\sum_{i=1}^{n} c^i \upsilon^i \leq t, \tag{2}$$

and the non-negativity constraints t, $1 - t \geq 0$, $c^i \geq 0$ and $\upsilon^i > 0$, for all i = 1, 2, . . . , n. Note that the term $\sum_{i=1}^{n} c^i$ in the utility function embodies the assumption that all consumption goods are perfect substitutes for each other. Note also that the consumer's decision problem described above is a familiar use of the theory of household production. The consumer buys the consumption good and then uses her own time to turn it into a form that generates utility. The time constraint (2) shows that merely buying the consumption good from a shop is not enough; household time must be applied.

It should also be pointed out that leisure time t is important not only because consumption is impossible without it, but also because it enters the utility function directly. This latter role of leisure is, however, not crucial to the results derived in this paper. In fact, all the results of this paper can also be proved for the simpler utility function $U(\sum_{i=1}^{n} c^i, t) = \sum_{i=1}^{n} c^i$.

It will be assumed that the varieties that are lower in quality are cheaper as well. In other words, it is assumed that $F(K, L;\upsilon)$ is increasing in υ or, equivalently, that $p(\omega_K, \omega_L, \upsilon)$ is decreasing in υ (i.e., $p_\upsilon \equiv \partial p/\partial\upsilon < 0$ if $p(\omega_K, \omega_L, \cdot)$ is differentiable). For simplicity, it will be further assumed that $p(\omega_K, \omega_L, \cdot)$ is strictly convex (i.e., $p_{\upsilon\upsilon} > 0$ if $p(\omega_K, \omega_L, \cdot)$ is twice differentiable) and that $\lim_{\upsilon \to 0} p(\omega_K, \omega_L, \upsilon) = \infty$.

Under these conditions it can be shown that both constraints, (1) and (2), are binding at the optimum values of the choice variables. Consequently, these equations together imply

$$\sum_{i=1}^{n} c^i \left[p(\omega_K, \omega_L, \upsilon^i) + \omega_L h_j \upsilon^i \right] = \omega_L h_j + \omega_K k. \tag{3}$$

The assumption that the consumer goods are perefect substitutes can be shown to yield the result that the utility maximizing consumer will consume only one variety. That is, there is a υ such that the optimum choice obeys $\upsilon^1 = \upsilon^2 = . . . = \upsilon^n = \upsilon$. Therefore, $c \equiv \sum_{i=1}^{n} c^i$ denotes the amount consumed of variety υ. Note that the optimal values of c, υ and t—which I will denote c_j, υ_j and t_j—must maximize $U(c, t)$ subject to

$$cp(\omega_K, \omega_L, \upsilon) = \omega_L h_j (1 - t) + \omega_K k, \tag{4}$$

$$c\upsilon = t, \tag{5}$$

and $c,\ t,\ 1 - t \geq 0$ and $\upsilon > 0$.

In the interior solution case of $0 < t_j < 1$, the first order condition is

$$\frac{U_t(c_j,\ t_j)}{U_c(c_j,\ t_j)} = \frac{p_\upsilon(\omega_K,\ \omega_L,\ \upsilon_j) + \omega_L h_j}{p(\omega_K,\ \omega_L,\ \upsilon_j) - \upsilon_j p_\upsilon(\omega_K,\ \omega_L,\ \upsilon_j)},$$

where $U_t(c,t) \equiv \partial U(c,t)/\partial t$ and $U_c(c,t) \equiv \partial U(c,t)/\partial c$ are the marginal utilities of leisure and consumption, respectively.

It will be assumed that $U(\cdot,\ \cdot)$ is homothetic. This implies that the marginal rate of substitution $U_t(c,t)/U_c(c,t)$ is a function of t/c or, by (5), υ. Therefore, if we denote the marginal rate of substitution by $m(\upsilon)$, the first-order condition may be rewritten as

$$m(\upsilon_j) = \frac{p_\upsilon(\omega_K,\ \omega_L,\ \upsilon_j) + \omega_L h_j}{p(\omega_K,\ \omega_L,\ \upsilon_j) - \upsilon_j p_\upsilon(\omega_K,\ \omega_L,\ \upsilon_j)}. \tag{6}$$

(Note that if we use the simpler utility function $U(\sum_{i=1}^{n} c^{i},\ t) = \sum_{i=1}^{n} c^{i}$, which implies $U_t(c,t) = 0$, then the first-order condition reduces to $p_\upsilon(\omega_K,\ \omega_L,\ \upsilon_j) + \omega_L h_j = 0$.)

For some values of the parameters it may happen that $t_j = 1$, in which case c_j and υ_j are obtained from (4) and (5) with 1 substituted for t. This case in which the consumer never leaves home, could occur when capital income $\omega_K k$ is so large that going to work is pointless. In this paper such a possibility will be assumed away and this corner solution will not be discussed any further.

Note that $\partial\upsilon_j/\partial k = 0$. That is, υ_j does not depend on k. Thus, as (5) suggests, if k increases both c_j and t_j will increase but c_j/t_j will not change. This implies that all citizens of j, irrespective of their capital endowments, will consume the variety υ_j that is determined by (6). It can also be shown that (6) implies that $\partial\upsilon_j/\partial h_j < 0$ under the assumptions $p_\upsilon < 0$, $p_{\upsilon\upsilon} > 0$ and a quasi-concave and homothetic $U(\cdot,\ \cdot)$. So, other things equal, a person with higher ability will consume a superior variety. An important effect of this is that if goods are actually traded in free trade equilibrium with factor price equalization (FTEWFPE), the high ability country must import the high quality good from, and export the low quality good to, the low ability country. Finally, it is straightforward to show that $\partial c_j/\partial h_j > 0$; consumption levels increase with ability.

Now if C_j is the total amount of υ_j consumed by the citizens of j, and if L_j is the total amount of effective labor time supplied by the citizens of j, and if T_j is the total leisure enjoyed by the citizens of j, then (4) and (5) imply

$$C_j p(\omega_K,\ \omega_L,\ \upsilon_j) = \omega_L L_j + \omega_K K_j \tag{7}$$

$$C_j \upsilon_j = N_j - \frac{L_j}{h_j} \equiv T_j. \tag{8}$$

Moreover, (7) and (8) together imply the country j counterpart of (3):

$$C_j \left[p(\omega_K,\ \omega_L,\ \upsilon_j) + \omega_L h_j \upsilon_j \right] = \omega_L h_j N_j + \omega_K K_j. \tag{9}$$

Equations (6)-(8) summarize, in terms of aggregates, the model's demand side. The comparative statics results mentioned in the paragraph before last translate into the following proposition.

Proposition 1. *$\partial\upsilon_j/\partial K_j = 0$, $\partial C_j/\partial K_j > 0$, $\partial L_j/dK_j < 0$, $\partial\upsilon_j/\partial h_j < 0$, $\partial C_j/\partial h_j > 0$.*

B. MARKET CLEARING CONDITIONS

Using Shephard's Lemma, let

$$a_{lj} \equiv a_l(\omega_K,\ \omega_L,\ \upsilon_j) \equiv \frac{\partial p(\omega_K,\ \omega_L,\ \upsilon_j)}{\partial \omega_l} \tag{10}$$

be the amount of resource l, where $l = K, L$, used in the production of one unit of good υ_j. Also, let X_{ji} denote the amount of υ_i that is produced in country j, and let $l^x_{ji} = a_{li} \cdot X_{ji}$ denote the amount of resource l used in country j to produce υ_i. Then the market clearing condition for resource l in FTEWFPE is

$$l^x_{j1} + l^x_{j2} = l_j, \tag{11}$$

for $j = 1,2$ and $l = K, L$. Similarly, the market clearing condition for υ_j is

$$X_{1j} + X_{2j} = C_j, \tag{12}$$

for $j = 1,2$. This condition, which implies that country j's demand for good υ_j must be equal to the worldwide output of υ_j, can be shown to ensure balanced trade. Equations (11) and (12) together imply

$$a_{l1}C_1 + a_{l2}C_2 = l_1 + l_2, \tag{13}$$

for $l = K, L$.

Equations (6), (9) and (13) can be used to solve for the FTEWFPE of the two-country world z. Since not all nominal prices can be uniquely solved for, let us assume $\omega_K = 1$. Equation (6) then solves for υ_j in terms of ω_L. Then (9) solves for C_j again in terms of ω_L. Finally, (13) for the case of $l = K$ solves for ω_L and closes the model. Once ω_L is determined one can go back to either (7) or (8) and determine L_j as well. Then (10)-(12) can be used to solve all other variables.

Now, recall that to simplify matters, $p(\upsilon)$, the price of good υ, was assumed, earlier in this section, to be equal to the unit cost $p(\omega_K, \omega_L, \upsilon)$ whether or not υ is produced in equilibrium. Note that $p(\upsilon) < p(\omega_K, \omega_L, \upsilon)$—but not $p(\upsilon) > p(\omega_K, \omega_L, \upsilon)$—is consistent with perfect competition if υ is not produced in equilibrium. However, my simplifying assumption is not restrictive in the sense that if there is a FTEWFPE such that $p(\upsilon) < p(\omega_K, \omega_L, \upsilon)$ for some υ not produced in the FTEWFPE, then there will also be an otherwise identical FTEWFPE in which $p(\upsilon) = p(\omega_K, \omega_L, \upsilon)$ for that υ.

Finally, to ensure that the solution for X_{ji} is non-negative, the capital-labor ratios of both countries must lie within the so-called "cone of diversification". Formally, in any FTEWFPE

$$\min\left\{\frac{a_{K1}}{a_{L1}}, \frac{a_{K2}}{a_{L2}}\right\} \leq \frac{K_j}{L_j} \leq \max\left\{\frac{a_{K1}}{a_{L1}}, \frac{a_{K2}}{a_{L2}}\right\} \tag{14}$$

must hold for $j = 1,2$.

To summarize, in a FTEWFPE all agents act as price taking maximizers, factor markets clear within each country, the worldwide goods markets are balanced, and the price of any commodity—product or factor—is the same all over the world.

3. THE RESULTS

This section describes the main features of trade under the assumptions of this paper. I will argue that my model can explain several empirically well established features of North-South trade that cannot be explained by the HOV model. The proofs of the results are straightforward and are outlined in the next section. For reasons of space, the formal proofs have been collected in an appendix that is available directly from the author.

Recall that $z = (N,K,h)$—where $N = (N_1, N_2)$, $K = (K_1, K_2)$, and $h = (h_1, h_2)$—denotes a two-country world.

Theorem 1. *There exist z for which a free trade equilibrium with factor price equalization* (*FTEWFPE*) *exists such that* $\upsilon_1 \neq \upsilon_2$.

Proving Theorem 1 amounts to proving that there exists z with $h_1 \neq h_2$ such that a FTEWFPE exists, for in that case Proposition 1 would imply $\upsilon_1 \neq \upsilon_2$. For concreteness, I will assume $h_1 < h_2$ for the rest of this paper and it will, therefore, be convenient to identify country 1 with the South and country 2 with the North. Theorem 1 then implies, by way of Proposition 1, that $\upsilon_1 > \upsilon_2$. Time, which is necessary in consumption, will have a higher value in country 2 and, therefore, the citizens of country 2 will choose higher quality goods that allow higher levels of consumption per unit of time.

While Theorem 1 is in contradiction to the underlying spirit of HOV, in which consumption patterns are identical everywhere, it is consistent with the empirically well established international differences in consumption patterns. Markusen (1986) showed how, under preferences characterized by a special form of non-homotheticity, consumption pattern differences could be explained. Theorem 1 provides an alternative explanation that retains the traditional assumption of identical and homothetic preferences, and adds Becker's theory of the allocation of time and international differences in productivity. In other words, Theorem 1 broadens the range of circumstances under which international differences in consumption patterns can be explained.

For the rest of this paper I will assume that higher quality goods are more capital intensive than lower quality goods. Then, the fact that the North consumes a superior variety compared to South implies that North's imports, if any, must be more capital intensive than its exports. (This is what Leontief (1953) had found in US data for 1947 as was pointed out in the Introduction. Although the methods used by Leontief have since been refined, especially by Leamer (1980), the empirical difficulty associated with HOV that Leontief had discussed has not gone away.) Whether this trade pattern is consistent with HOV would depend on the capital-labor ratios in countries 1 and 2. Under this paper's assumptions, capital K_j and population N_j and, therefore, the per capita capital stock K_j/N_j are all exogenous. But the capital-labor ratio K_j/L_j is endogenous because L_j is endogenous. There would in general exist z such that in FTEWFPE the capital-labor ratio is higher in country 1, the South. In such cases trade would be just as under HOV because the country with the higher capital-labor ratio would be exporting the capital intensive good and vice versa.

Theorem 2. (*HOV*) *There exist z for which a free trade equilibrium with factor price equalization* (*FTEWFPE*) *exists such that the capital-labor ratio is higher in country 1* (*i.e.*, $K_1/L_1 > K_2/L_2$), *the amounts traded are positive* (*i.e.*, $X_{12}, X_{21} > 0$), *and the good consumed and imported by country 2 is more capital intensive than the good consumed and imported by country 1* (*i.e.*, $a_{K1}/a_{L1} < a_{K2}/a_{L2}$).

Note that since the South's exports must *necessarily* be the capital intensive good that the North consumes, trade under this paper's assumptions can be consistent with HOV only if, atypically, the low ability country (South) has the higher capital-labor ratio. In the more typical case, the capital labor ratio would be lower in country 1, the South. And in this case, trade would have to go against the HOV prediction.

Theorem 3. (*Leontief Paradox*) *There exist z for which a FTEWFPE exists such that the capital-labor ratio is lower in country 1* (*i.e.*, $K_1/L_1 < K_2/L_2$), *the amounts traded are positive* (*i.e.*, $X_{12}, X_{21} > 0$), *and the good consumed and imported by country 2 is more capital intensive than the good consumed and imported by country 1* (*i.e.*, $a_{K1}/a_{L1} < a_{K2}/a_{L2}$).

Recall that Leontief (1953) had argued that US imports had a higher capital to labor ratio than US exports even though the US was capital abundant. Leontief went on to suggest that if labor quality differences were noted, the US would instead be relatively abundant in labor (i.e., effective labor) and the 'paradox' would be resolved. Trefler (1993) produces empirical support for Leontief's view. In the Theorem 3 scenario, however, the HOV trade pattern does not hold even though the discussion is wholly in terms of capital and *effective* labor. It should be pointed out that Trefler's modified HOV still is unable to explain the international variation in consumption patterns and the low levels of North-South trade. The changes introduced in this paper to address those issues result in the dilution of the HOV prediction on the pattern of trade.

Note that in the Theorem 2 scenario the country with the higher (resp. lower) capital-labor ratio consumes the capital (resp. labor) intensive good. Therefore, by Rybczynski's theorem, each country's production would be skewed towards the one good that is consumed by its citizens (i.e., $X_{22}/X_{21} > X_{12}/X_{11}$). In the extreme case, this skewing may in fact be so sharp that each country might produce only the good that is domestically consumed and there may, therefore, be no trade. And what is important to note is that this absence of trade would coexist with large differences in the capital-labor ratios of the two countries. While this is actually typical of North-South trade, it is impossible under HOV assumptions.

Theorem 4. (*North-South Trade*) *There exist z for which there are FTEWFPE such that countries 1 and 2 have different capital-labor ratios (i.e.,* $K_1/L_1 \neq K_2/L_2$) *and there is no trade* (*i.e.,* $X_{12} = X_{21} = 0$). *Moreover, the country with the higher capital-labor ratio will also be the country that consumes the higher quality good and its citizens will have a higher level of ability* (*or human capital*) *than the other country. If higher quality goods are assumed to be more capital intensive than lower quality goods, the country with the higher capital-labor ratio will also be the country with the higher per capita income.*

Note that not only does Theorem 4 explain how low levels of trade could coexist with large differences in capital-labor ratios, it also links such trade to North-South trade. Had it instead been the case, for instance, that the country with the higher capital-labor ratio is necessarily the *poorer* of the two countries, the explanation of low levels of trade between countries 1 and 2 in Theorem 4 would not have been relevant to the issue of low levels of North-South trade.

Now since there are z such that $K_1/L_1 > K_2/L_2$ (Theorem 2) and z such that $K_1/L_1 < K_2/L_2$ (Theorem 3), it is no surprise that there are z such that $K_1/L_1 = K_2/L_2$. Under HOV there would be no trade in this case because the only source of comparative advantage in HOV are the international differences in capital-labor ratios. In this paper's model, however, international differences in worker ability or human capital are an independent source of comparative advantage, and trade can take place even if $K_1/L_1 = K_2/L_2$. In fact, if $K_1/L_1 = K_2/L_2$, then by Rybczynski's Theorem, not only would each country produce both goods, the relative composition of output would be the same in the two countries in the sense that X_{j2}/X_{j1} would be the same for $j = 1,2$. Since country j consumes only one good υ_j, trade must occur.

Theorem 5. *There exists z such that there is a FTEWFPE for z in which* $K_1/L_1 = K_2/L_2$ *and the amounts traded are strictly positive* (*i.e.,* $X_{12}, X_{21} > 0$).

4. AN OUTLINE OF THE PROOFS

The formal proofs of Theorems 1-5 are available from the author. This section outlines the logic underlying those proofs. In the proofs, I make use of the concept of Integrated Equilibrium. The competitive equilibrium of an integrated world in which all produced goods are freely traded across countries and all factors of production are perfectly mobile between countries is referred to as the Integrated Equilibrium (IE).

(For more on this equilibrium concept, see Dixit and Norman 1980 and Helpman and Krugman 1985.) In the IE, (6)-(9) clearly must hold. Similarly, (13) must hold. Once the normalization $\omega_k = 1$ is adopted, (6), (9), and (13) can be used to determine the IE value of ω_L exactly as they were used, in section 2, to determine the value of ω_L in the Free Trade Equilibrium with Factor Price Equalization (FTEWFPE). Once ω_L is determined the remaining unknown L_j can be solved for from (7) or (8).

It is obvious that the FTEWFPE of z is indistinguishable from the IE of z even though capital and labor are immobile in the FTEWFPE and mobile in the IE. (It will be assumed throughout that the conditions for uniqueness of equilibria are satisfied.) Moreover, it can be shown that if the IE of z satisfies the "cone of diversification" requirement (14), then z has a FTEWFPE that is indistinguishable from the IE of z.

Let the value of a variable in the integrated equilibrium for z be denoted by the symbol for that variable followed by z in parentheses; e.g., let $\omega(z) \equiv (\omega_L(z), \omega_K(z) \equiv 1)$ denote the *normalized* factor prices that prevail in the integrated equilibrium for z.

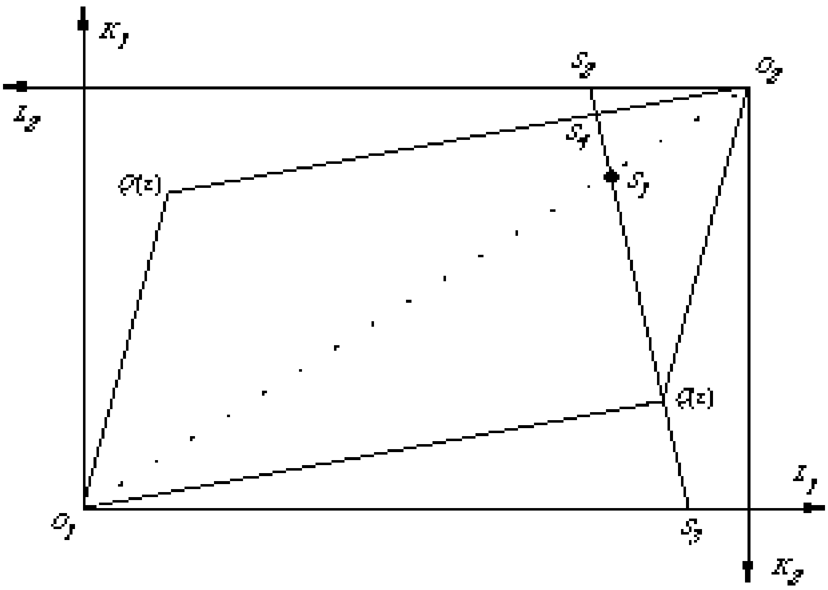

Figure 1. Trade with Factor Price Equalization

Figure 1 shows an Edgeworth Box representing the IE of z. The box's dimensions are $K_W = K_1 + K_2$ and $L_W(z) = L_1(z) + L_2(z)$. Traditionally Edgeworth Boxes have been used in models in which both capital and labor are inelastically supplied. It will be shown, however, that the Edgeworth Box analysis continues to be useful in this paper's model even though $L_W(z)$ is endogenously determined.

Let $l_i^x(z) = l^x_{1i}(z) + l^x_{2i}(z) = a_{li}(z)C_i(z)$ be the amount of resource l used worldwide in the production of $C_i(z)$ units of good $\upsilon_i(z)$, for $l = K,L$ and $i = 1,2$, and let $V_i^x(z) = (K_i^x(z), L_i^x(z))$. Also, let $V_j^s(z) = (K_j, L_j(z))$ denote the factors sold by the citizens of country j. In the IE we must have $\sum_{i=1}^{2} V_j^x(z) = \sum_{j=1}^{2} V_j^s(z) = (K_W(z), L_W(z))$. Therefore, one can find points $Q(z)$ and $Q'(z)$ in the Edgeworth Box such that $O_1Q(z) = O_2Q'(z) = V_1^x(z)$ and $O_jQ'(z) = O_2Q(z) = V_2^x(z)$, and one can find a point $S(z)$—not shown—in the box such that $O_1S(z) = V_j^s(z)$, $j = 1,2$. That is, $O_1Q(z)$ (resp. $O_2Q(z)$) represents the quantities of K and L used in the manufacture of the variety $\upsilon_1(z)$ (resp. $\upsilon_2(z)$) in the IE, and $O_1S(z)$ (resp. $O_2S(z)$) represents the quantities of K and L sold by the citizens of country 1 (resp. country 2) in the IE.

My assumptions that $h_1 < h_2$ and that superior goods are more capital intensive than inferior goods implies that $Q(z)$ and $Q'(z)$ must be below and above the diagonal, respectively.

Now, by definition, $p(\omega_K, \omega_L, \upsilon_j) = \omega_K a_{Kj} + \omega_L a_{Lj}$. Then (7) implies $\omega_K(z)K_j^x(z) + \omega_L(z)L_j^x(z) = \omega_K(z)K_j + \omega_L(z)L_j(z)$. That is, the market values of the resource bundles $V_j^x(z)$ and $V_j^s(z)$ must be equal. Therefore, the slope of the line through $S(z)$ and $Q(z)$ must be $-\omega_L(z)/\omega_K(z)$. In figure 1 this line is S_2S_3. Note that $Q(z)S(z)$ (resp. $S(z)Q(z)$) represents the factor content of country 1's (resp. country 2's) exports, which is $[K_1 - K_1^x(z), L_1(z) - L_1^x(z)]$ (resp. $[K_2 - K_2^x(z), L_2(z) - L_2^x(z)]$).

To see how the discussion of IE, in which K and L are mobile, helps in the analysis of FTEWFPE, in which K and L are not mobile, recall that the IE and FTEWFPE are indistinguishable if the "cone of diversification" requirement (14) is satisfied in IE. This requires that $O_jS(z)$ must lie between $O_jQ(z)$ and $O_jQ'(z)$, which requires that $S(z)$ must lie on $Q(z)S_4$. In general, the IE of z need not be a FTEWFPE, in which case $S(z)$ would be on S_2S_3 but not on $Q(z)S_4$. However, it can be shown that even if $S(z)$ is not on $Q(z)S_4$, one can find z' such that the Edgeworth Box representing the IE of z' would be identical to the one for z except that $S(z')$ would be on $Q(z)S_4$. Once again, the formal statement and proof of this result—let's call it Lemma 1—is available directly from the author. Lemma 1 is crucial because it shows that if one takes an arbitrary point S' on S_2S_3, one can construct z' such that the Edgeworth Box that represents the IE of z' would be the same in every detail as the one for z in figure 1 except that $S(z')$ would now coincide with S'. This is significant in the sense that the proofs of Theorems 1—5 then follow automatically.

To prove Theorem 2 one can pick S' on S_1S_4 and, following Lemma 1, construct z' such that $S(z')$ coincides with S'. Since $S(z')$ is on $Q(z)S_4$ the IE of z' is the same as its FTEWFPE. And in this FTEWFPE country 1 has the higher capital-labor ratio, since S' is above the diagonal, and exports the capital intensive good. To prove Theorem 3 one can pick S' on $Q(z)S_1$ and do as before. In this case country 1 would have the smaller capital-labor ratio and export the capital intensive good. To prove Theorem 4 one can let S' coincide with $Q(z)$ and do as before. In this case there would be no trade in the FTEWFPE despite international differences in capital-labor ratios. To prove Theorem 5 one can pick S' to coincide with S_1. In this case there would be trade in the FTEWFPE despite equal capital-labor ratios in the two countries. And as for Theorem 1, it should be clear that its proof is contained in each of the other proofs.

5. CONCLUSION

The pure theory of international trade has highlighted several factors that can plausibly be said to be the sources of gains from trade. These include international differences in relative factor proportions, differences in technology, differences in government policies, increasing returns to scale, and imperfect competition. This paper has argued that differences in labor skills can also be a source of gains from trade. One obvious extension of this paper would be to test for the importance of labor income differences in trade.

It would also be interesting to work out models that retain this paper's view of quality but introduce elements from other existing models of trade with vertical product differentiation. One could introduce a homogeneous good, overlapping income distributions for the trading countries (possibly several of them instead of two), and/or a one-factor technology such as the one in Stokey.

Finally, it may be useful to end by pointing out that this paper's argument in favor of differences in worker ability as a source of gains from trade should not be seen as an argument against the other sources of gains from trade that have been discussed in the literature. Echoing the title of Markusen (1986), it is as important now as it ever has been to be 'eclectic'.

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