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Economic Growth with Negative Externalities in Innovation*

It is argued that the linearity of the R&D sector's technology in Paul Romer's model of endogenous technological progress is very restrictive. An externality, stemming from the simultaneity and overlap in researchers' activities, is modeled into the R&D sector. The optimality analysis of this model leads to a sceptical view of the case for government subsidies for investments in R&D. The quality of human capital, a subject discussed rarely if ever in theories of endogenous technological progress, is shown to affect the model economy's long-run growth rate.

1. Introduction

In most modern economies private business enterprises regularly invest a great deal of money in research and development.¹ And yet, until recently, most theories of long-term economic growth had ignored this phenomenon and had modeled technological progress either as exogenous (Solow 1956; Mirrlees 1967), or as the happy by-product of activities undertaken for other reasons (Arrow 1962). This gap in the literature was largely filled in the early 1990s, primarily by Romer (1990).² In Romer's model, the engine of growth is fueled by the innovative activities of profit maximizing firms.

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¹The number of scientists and engineers employed by business enterprises for research purposes approached 600,000 in 1985 in the United States, and 280,000 in 1988 in Japan. OECD countries spent \$155 billion on industrial research in 1985. Such spending has been rising in the OECD countries, both as a share of GDP and as a share of gross fixed capital formation. The latter share was 21% in 1985 in the United States. For further details, see Grossman and Helpman (1991).

²Other important and early contributions include Judd (1985), Segerstrom, Anant, and Dinopoulous (1990), and a series of papers by Gene Grossman and Elhanan Helpman discussed in Grossman and Helpman (1991).

This paper presents a model of long-run growth that includes Romer's model as a special case. It is shown that an important normative result of Romer's model does not generalize. Romer (1990) had argued that the private sector's investment in R&D is less than the socially optimal level and that such expenditure should therefore be subsidized by the government. In this paper, however, overinvestment is possible. It is also shown that there is a positive relation between the *quality* of research labor and long-run growth. Unlike most theories of endogenous technological progress, labor quality and labor quantity are not perfectly substitutable in this paper.

In setting up a model of technological progress, it might appear quite natural to assume that, other things equal, an economy with, say, 200 researchers would have twice the rate of technological progress (measured by the number of patents awarded per unit time) as would an economy with 100 researchers. Indeed, this linearity is a prominent feature of all recent theories of endogenous growth, including Romer (1990) and the theories surveyed in Grossman and Helpman (1991). It should be noted, however, that this assumption implies that for a large enough pool of researchers, any number of inventions, no matter how large, could be achieved in any period of time, no matter how small. For example, all inventions made from, say, 1632 to 1994 could have been worked out in just the first week of 1632 if only there had been enough researchers back then. And those researchers would only have had to be knowledgeable about the state of the art as it would have been in 1632. With that knowledge they would have worked *everything* out, in just one week. This possibility would strike most people as far-fetched, not because of any doubts about whether a large enough group of researchers could ever be assembled, but because of strong doubts about whether this startling feat could be achieved by *any* finite number of researchers. The mere possibility of unbounded rates of invention condemns linearity as unrealistic. Boundedness—or, more generally, diminishing returns—in the research sector technology would seem to be the only plausible alternative.

Under diminishing returns, an increase in the number of researchers leads to a less than proportionate increase in the number of inventions. This implies a fall in the average output of inventions per researcher. The patent race literature (see Reinganum 1989 for a survey) gives some indications as to how this could happen. At any given moment the number of research projects that are both potentially lucrative and seemingly tractable is limited. This leads to patent races. In such races, when one researcher announces that she has solved the problem, the others have to abandon their work and start all over again on some other problem. The expected amount of effort lost in this way by a researcher depends upon how popular the problems being investigated by this researcher are with other researchers. If the num-

ber of researchers doubles for some reason—perhaps because of a diversion of labor away from other areas and into research—and if the number of research projects fails to double, then the number of researchers *per research project* would increase. There would, therefore, be greater “crowding” or “congestion.” This would reduce the success rate of the average researcher, and, therefore, the aggregate success rate would fail to double. Thus, the congestion externality introduces diminishing returns, of which boundedness is a special case, into the research sector. In Sections 2 and 3 below, it is shown that the congestion externality just described can be used to construct a Romer-type growth model that does not have linearity in its research technology.

One should note that the congestion externality is better referred to as an institutional externality rather than a technological externality. The presence of rivals does not reduce the ability of a researcher to solve a given research problem; it only affects her chances of being the first to solve that problem. So, it is the “first past the post” nature of a patent based system that is at the root of the congestion externality.

The addition of this externality to the Romer model also alters its optimality analysis. (This is hardly surprising; one would expect such an outcome whenever a new externality is added.) In Romer (1990) the equilibrium level of investment in R&D is less than the socially optimal level. But the congestion externality of this paper makes overinvestment a possibility. This is because each researcher does her work without considering the fact that her success would be someone else’s failure.

The congestion externality is also related to the *quality* of human capital, a topic rarely discussed in the growth literature. The externality comes into play when a large number of researchers pursue the same research project (or, similar research projects). But if the researchers are original enough to try to identify and explore research ideas that are relatively unique, then congestion would be less of a problem. Thus, a direct relation can be established between the quality of the researchers and economic growth.

The rest of the paper is organized as follows. Section 2 discusses the nature of the dependence of a researcher’s work on concurrent research done by others. Section 3 analyzes the balanced growth equilibrium of an economy with the “crowding” externality, and Section 4 discusses optimality issues. Section 5 concludes the paper.

2. The Research Sector

Let there be m firms in the research sector, indexed by $k = 1, 2, \dots, m$. These firms hire labor to do research. Researchers come up with blue-

prints for intermediate goods that would be useful to producers. The patents obtained on the inventions are then sold for profit.

Researchers are assumed to work together in groups because collaboration is important in research. The optimal size for such research groups is assumed to be a finite number, s ; the total output of the L_k workers of firm k would, therefore, be maximized if and only if they work in groups of size s . Given these assumptions, a group of n firms, each with s workers, would be essentially the same as one firm with ns workers. Putting it a little differently, if the research firms make their employees work in groups of size s —as indeed they should—the total output of inventions by the research sector would depend on L_A , the total number of researchers employed in the research sector, and not on how L_A is distributed among the various research firms. Moreover, if firm k employs L_k workers, its output would be L_k/L_A times the industry output.

Let A denote the number of intermediate goods that exist currently. An increase in A represents an increase in the technological options that are available to producers. A is also a proxy for the level of knowledge currently available, and increases in A are, therefore, assumed to increase the productivity of researchers.

The total output of the research sector is, therefore, given by a function $F(A, L_A)$, and the output of firm k is given by $F(A, L_A) \cdot L_k/L_A$. Letting $F(A, L_A) = Af(L_A)$ and adding a discontinuity, for reasons to be given below, at $L_k = 1$, the number of patents earned per unit time by firm k is assumed to be

$$a_k = \begin{cases} A \frac{f(L_{-k} + L_k)}{L_{-k} + L_k} L_k, & \text{if } L_k \geq 1 \\ 0, & \text{if } 0 \leq L_k < 1, \end{cases} \quad (1)$$

where L_{-k} is the amount of labor employed by all firms other than k ; clearly, $L_A = L_k + L_{-k}$. It is also assumed that $f(0) = 0$, $f' \geq 0$, and $f(L_A)/L_A$ is non-increasing in L_A . When $f(L_A)/L_A$ is constant (e.g., when $f(L_A) = \delta L_A$) this paper's model reduces to the Romer model. This paper will, therefore, concentrate on the case in which $f(L_A)/L_A$ is decreasing in L_A . Assuming $L_k \geq 1$, firm k 's average product is $a_k/L_k = Af(L_{-k} + L_k)/(L_{-k} + L_k)$. Therefore, if $f(L_A)/L_A$ is decreasing in L_A , a_k/L_k must be decreasing in L_k , when L_{-k} is taken as given.³

The discontinuity assumed in α_k implies that in equilibrium $L_k \geq 1$

³The units of labor can be so chosen that the optimal group size is unity. Also, for large enough L_A and L_k there is no significant difference between labor being discrete or continuous.

must be true as long as L_k is not zero. The discontinuity is needed to ensure the existence of a perfectly competitive equilibrium with free entry (to be defined later) in the research sector. Discontinuities of this sort are usually associated with the existence of indivisibilities.

Note that as long as $L_{kt} \geq 1$ for all k at time t , from now on the subscript t will indicate the value of a variable at time t , the rate of *aggregate* technological progress, or $\dot{A}_t \equiv dA_t/dt$, is given by

$$\dot{A}_t = \sum_{k=1}^{m_t} a_{kt} = A_t f(L_{At}). \quad (2)$$

The assumption that $f(L_A)/L_A$ is decreasing in L_A implies that \dot{A}_t is subject to diminishing returns, and could even be bounded, in L_{At} . Similarly, the rate of growth of technology, $g_{At} = \dot{A}_t/A_t = f(L_{At})$, could be bounded in L_{At} .

Diminishing returns makes sense because the alternative, that is, non-diminishing returns, has absurd implications. In Romer's model $\dot{A}_t = \delta A_t L_{At}$. This implies that X inventions can be realized in Y units of time, no matter how large X is or how short Y is, if only $X/\delta A Y$ units of human capital are available.⁴ As was argued in the introduction, this is unrealistic, which is why this paper incorporates diminishing returns to L_A .

It can also be argued, however, that diminishing returns is the natural consequence of certain important features of the research process. Note that an important property of (1) is that if $f(\cdot)$ has either increasing returns or decreasing returns, then firm k 's technology necessarily involves an externality. Firm k 's productivity is in those cases affected by changes in L_{-k} . Only if $f(\cdot)$ has constant returns (e.g., if $f(L_A) = \delta L_A$, as in Romer) does the externality disappear. This suggests that any "story" that one could tell to motivate non-constant returns would *have* to involve an externality. The reader will recall that such an externality was discussed in the introduction.⁵

3. The Balanced Growth Equilibrium

This section describes the equilibrium of a model of economic growth that follows Romer (1990) but uses the research sector technology (1) that

⁴Human capital is measured by the time spent in study. So, if the time needed to learn the state of the art (say, the time needed to get a Ph.D.) is relatively stable over time, and if that amount of time is normalized to unity, then L_A could be made to refer both to the amount of human capital and to the number of researchers knowledgeable in the state of the art.

⁵Note that diminishing returns implies the externality, and not the other way around. Take the case in which $f(L_A)/L_A$ is decreasing in L_A and $L_{-k} = 0$. In this case all research is being done by one giant firm k and, therefore, no externality is apparent in the giant firm's technology $\alpha_k = Af(L_A) = Af(L_k)$. However, note that α_k is still subject to diminishing returns.

has, not constant returns as in Romer, but diminishing returns. The next section will calculate the optimal allocation and compare it with this section's equilibrium allocation. The aim is to see whether Romer's conclusion that there is underinvestment in research, continues to hold if his linearity assumption is relaxed. To be able to do this satisfactorily, however, this paper will, for technical reasons to be discussed later, use a modification of the Romer model that was adopted in chapter 3 of Grossman and Helpman (1991).

The Consumer

The representative consumer chooses the time path c_t of the amount consumed of the final good to maximize

$$U_0 = \int_0^\infty e^{-\rho t} \ln c_t dt , \quad (3)$$

subject to an intertemporal budget constraint, which requires that the discounted sum of lifetime consumption expenditures not exceed the discounted sum of the incomes from all factors of production, with all prices taken as given. This yields the Euler Equation (see Blanchard and Fischer 1989, section 2.2)

$$r_t = \rho + \frac{\dot{c}_t}{c_t} , \quad (4)$$

where r_t is the real interest rate on consumption loans, and $\dot{c}_t \equiv dc_t/dt$ is the rate of change of consumption.

The Production of the Consumption Good

It is assumed that c_t units of the consumption good are produced at time t with L_{ct} units of labor and x_{jt} units of the intermediate good j , for all j in the interval $(0, A_t)$, where A_t , as was discussed in Section 2, represents the number of intermediate goods invented up to date t , and is, therefore, a proxy for the state of the art. The production function, made famous by Dixit and Stiglitz (1977), is⁶

$$c_t = L_{ct}^{1-\alpha} \int_0^{A_t} x_{jt}^\alpha dj , \quad (5)$$

with $0 < \alpha < 1$.

⁶As Romer (1990) makes clear, increases in A_t are equivalent to Harrod neutral technological progress.

Producers maximize profits taking all prices as given. So, if w_t is the wage rate, measured in units of the consumption good, then w_t must be equal to the marginal product of labor:

$$w_t = (1 - \alpha)L_{ct}^{-\alpha} \int_0^{A_t} x_{jt}^\alpha dj . \quad (6)$$

(Note that in modeling a competitive industry with a constant returns technology, one can, without loss of generality, take the number of firms to be one.)

Also, if p_{jt} is the price of intermediate good j at time t , once again measured in units of the consumption good, then by the same reasoning

$$p_{jt} = \alpha L_{ct}^{1-\alpha} x_{jt}^{\alpha-1} . \quad (7)$$

The Production of Intermediate Goods

Each intermediate good $j \in (0, A_t)$ must obviously have been invented at some date prior to t . The producer of intermediate good j , it will be assumed for simplicity, has purchased a perpetual patent on good j from the firm that invented it. Each unit of an intermediate good is assumed to require one unit of labor to be produced. Thus, if L_{xt} is the amount of labor allocated to the production of intermediate goods at time t , then

$$\int_0^{A_t} x_{jt} dj = L_{xt} . \quad (8)$$

Note also that the objective of the monopolist producer of good j will be to maximize instantaneous profits $p_{jt}x_{jt} - w_t x_{jt}$ subject to the inverse demand (7).⁷ The first-order condition for a monopolist's profit maximization in this case yields

$$w_t = \alpha^2 L_{ct}^{1-\alpha} x_{jt}^{\alpha-1} = \alpha p_{jt} . \quad (9)$$

Equation (9) implies $x_{jt} = x_t$ and $p_{jt} = p_t$ for all j , and (8) then implies $x_t = L_{xt}/A_t$. Thus,

⁷Actually, the owner of a patent maximizes the discounted sum of the perpetual stream of profits. But since the price and output decisions at time t have no effect subsequently on either the demand for or the production costs of the intermediate good, intertemporal profit maximization boils down to maximizing instantaneous profit.

$$x_{jt} = x_t = L_{xt}/A_t \quad (10)$$

and

$$p_{jt} = p_t = \alpha L_{ct}^{1-\alpha} L_{xt}^{\alpha-1} A_t^{1-\alpha}. \quad (11)$$

Also, the profits from good j are

$$\pi_{jt} = \pi_t = (1 - \alpha)p_t x_t = \alpha(1 - \alpha)L_{ct}^{1-\alpha} L_{xt}^{\alpha} A_t^{-\alpha}. \quad (12)$$

With the substitution from (10), the wage rate in the consumption good sector in (6) becomes

$$w_t = (1 - \alpha)L_{ct}^{-\alpha} L_{xt}^{\alpha} A_t^{1-\alpha}, \quad (13)$$

and the wage rate in the intermediate goods sector in (9) becomes

$$w_t = \alpha^2 L_{ct}^{1-\alpha} L_{xt}^{\alpha-1} A_t^{1-\alpha}. \quad (14)$$

Full Employment in the Labor Market

If \bar{L} denotes the constant labor endowment in our economy, then full employment at each instant t requires

$$L_{At} + L_{xt} + L_{ct} = \bar{L}. \quad (15)$$

The necessary equality of wages in the consumption good and intermediate good sectors implies that $\alpha^2 L_{ct} = (1 - \alpha)L_{xt}$. This equation, when solved simultaneously with (15), yields

$$L_{xt} = \frac{\alpha^2}{1 - \alpha + \alpha^2} (\bar{L} - L_{At}), \quad (16)$$

$$L_{ct} = \frac{1 - \alpha}{1 - \alpha + \alpha^2} (\bar{L} - L_{At}), \quad (17)$$

at each t .

Research and the Invention of Intermediate Goods

Technological progress is represented in this paper by increases in A_t . From (5) and (10) we can write

$$c_t = A_t^{1-\alpha} L_{ct}^{1-\alpha} L_{xt}^{\alpha}. \quad (18)$$

Thus, bigger values of A_t imply bigger outputs of the consumption good for given values of L_{ct} and L_{xt} .

New intermediate goods are invented by research firms that employ labor. The research technology (1) was discussed in Section 2. (It is possible to build up a model that distinguishes between skilled, research labor [or human capital] and unskilled, production labor, but the results would not be significantly different.)

Patents for all existing intermediate goods can be traded at each t . In competitive markets, P_{Ajt} , the price at time t of the perpetual patent on intermediate good j , must be equal to the discounted sum of all profits from that good from t to infinity. Thus,

$$P_{Ajt} = \int_t^{\infty} e^{-[R_s - R_t]} \pi_{js} ds , \quad (19)$$

where $R_\tau = \int_0^\tau r_v dv$. Note that (12) implies that all existing intermediate goods must be worth the same; i.e., $P_{Ajt} = P_{At}$, for all $j \in (0, A_t)$.

At time t , the research firm k maximizes profits, $P_{At}a_k - w_t L_k$, taking as given the prices w_t and P_{At} , the technology (1), and L_{-k} , the labor employed in all research firms other than k . So, the research sector equilibrium is a Nash equilibrium in the sense that L_{-k} is taken as given, and it is a competitive equilibrium in the sense that w_t and P_{At} are taken as given. It will also be assumed that the free entry of firms into the research sector reduces profits to zero:

$$w_t = A_t \frac{f(L_{At})}{L_{At}} P_{At} . \quad (20)$$

Under this paper's assumptions on $f(\cdot)$ — $f(0) = 0$, $f' \geq 0$, and $f(L_A)/L_A$ non-increasing in L_A , one can show that there exists a zero-profit equilibrium such that $L_{kt} = 1$ for all firms k , and the number of firms is $m_t = L_{At}$, the total employment in research. If $f(L_A)/L_A$ is decreasing in L_A one can further prove that this equilibrium is unique.

Equilibrium

The various equilibrium conditions that have been derived so far can now be used to construct the equilibrium time path.

Specifically, it will be shown below that, if \bar{L}_A solves the equation

$$\frac{1 - \alpha(1 - \alpha)}{\alpha(1 - \alpha)} \frac{L_A}{f(L_A)} \rho + \frac{L_A}{\alpha(1 - \alpha)} = \bar{L} , \quad (21)$$

then there exists an intertemporal equilibrium in which $L_{At} = \bar{L}_A$, a constant, for all t .

Although the uniqueness of this equilibrium will not be proved, it can be shown, along the lines of Grossman and Helpman (1991, section 3.2), that the balanced growth equilibrium is also the only equilibrium.

If there is an equilibrium in which $L_{At} = L_A$, a constant, for all t , how would the other variables behave in that equilibrium? The (labor) market clearing conditions (16) and (17) imply that L_{xt} and L_{ct} must also take constant levels, L_x and L_c , which can be obtained by substituting $L_{At} = L_A$ in (16) and (17).

Further, if there is an equilibrium in which L_{At} and, therefore, L_{xt} and L_{ct} , are constant, then (18) would imply that the growth rates of c_t and A_t , $g_{ct} \equiv \dot{c}_t/c_t$ and $g_{At} \equiv \dot{A}_t/A_t$, must be related; $g_{ct} = (1 - \alpha)g_{At}$. Moreover, (2) would imply that g_{At} and, therefore, g_{ct} must be constant as well; $g_{At} = g_A = f(L_A)$, and $g_{ct} = g_c = (1 - \alpha)f(L_A)$.

The Euler equation (4) for the representative consumer's utility maximization problem must also hold. Therefore,

$$r_t = r = \rho + g_c = \rho + (1 - \alpha)f(L_A), \quad (22)$$

also a constant.

Also, (19) and (12) imply that the price of a patent must be

$$P_{At} = \alpha(1 - \alpha)L_c^{1-\alpha}L_x^\alpha \int_t^\infty e^{-r(s-t)} A_s^{-\alpha} ds. \quad (23)$$

Note that $g_{At} = f(L_A)$, a constant, implies $A_s = A_t \exp \{f(L_A)(s - t)\}$. If we make this substitution in (23) and substitute the resulting expression for P_{At} into (20), we get

$$w_t = \frac{f(L_A)}{L_A} \frac{\alpha(1 - \alpha)L_{ct}^{1-\alpha}L_{xt}^\alpha A_t^{1-\alpha}}{\rho + f(L_A)}. \quad (24)$$

Equating the wage in (24) with the wage in (13), or the wage in (14), and substituting out L_x and L_c using (16) and (17), we get (21), the condition that determines \bar{L}_A , the equilibrium level of research employment.

It is clear from the foregoing discussion that if $L_{At} = \bar{L}_A$ for all t , and if all equilibrium conditions are satisfied, then \bar{L}_A must solve (21). Or, put another way, if \bar{L}_A solves (21), then there is an equilibrium in which $L_{At} = \bar{L}_A$ for all t .

Note that (21) determines \bar{L}_A in terms of the parameters of the model,

and that every other variable can in turn be expressed in terms of \bar{L}_A , thus completing the description of the balanced growth equilibrium.

To summarize, the number of intermediate inputs A grows at the rate $g_A = f(\bar{L}_A) > 0$. Consumption grows at the rate $g_c = (1 - \alpha)g_A > 0$. Like L_A , L_x and L_c are constant. By (10), $g_x \equiv \dot{x}_t/x_t = -\alpha g_A < 0$; that is, the output of each intermediate good tends to zero because L_x is constant and A is growing at the rate g_A . The total output of all intermediate goods is, however, constant, since L_x is constant. By (11) and (14), $g_w = g_p = g_c > 0$. As technology improves, resources become more productive and, therefore, the real returns to these factors rise, too. By (12) and (20), the profits from any single intermediate good and the value of the patent on an intermediate good both grow at negative rates, $g_\pi = g_p = g_x < 0$, where $g_p \equiv (dP_{At}/dt)/P_{At}$. In totality, however, the intermediate goods sector holds its ground. Let $\hat{\pi} = A\pi$ be total profits from intermediate goods and let $\hat{P}_A = AP_A$ be the total value of all intermediate goods. It can be checked that the growth rates of $\hat{\pi}$ and \hat{P}_A are $g_{\hat{\pi}} = g_{\hat{P}} = (1 - \alpha)g_A = g_c > 0$. Recall that w , p , π , P_A etc. are all defined in units of the consumption good. Since labor income and profit income are growing at the same rate as consumption, relative factor income shares are being maintained at a constant level, as one would expect in a balanced growth equilibrium.

It is important to repeat that in this equilibrium $L_{At} = \bar{L}_A$ from $t = 0$ to $t = \infty$. That is, there are no transitional dynamics; the economy gets on the balanced growth path and stays on it thereafter. In Romer (1990), by contrast, transitional dynamics could potentially be important. In that model, there are two state variables, viz., physical capital, K , and technology, A . The balanced growth path is characterized by a unique value of K/A , and, except by coincidence, the initial conditions K_0 and A_0 will not be such that K_0/A_0 is equal to the balanced growth equilibrium value of K/A . This would make the transitional dynamics from the initial K_0/A_0 to the equilibrium K/A potentially important, especially for the normative evaluation of the equilibrium path. In this paper there is only one state variable, A , which does not reach or approach some specific value in equilibrium but instead grows continuously at the rate g_A . Thus there are no sources of *transitional* dynamics here. It will become clear in the next section that this feature of the model greatly facilitates the comparison of the equilibrium and optimal paths. (An earlier version of this paper based on Romer 1990 itself, and not on its Grossman-Helpman version, is available from the author.)

The comparative dynamics are pretty much the same as in Romer (1990). Under this paper's assumptions on $f(\cdot)$ — $f(0) = 0$, $f' \geq 0$, and $f'' \leq 0$, it can be shown that if the economy's endowment of labor \bar{L} increases, then so will \bar{L}_A , and, therefore, the growth rate $g_A = f(\bar{L}_A)$. (It should be kept in mind that \bar{L} refers to skilled labor that is capable of doing research.)

It can also be shown that greater impatience, i.e., a higher ρ , reduces \bar{L}_A and, therefore, $\bar{g}_A = f(\bar{L}_A)$.

Where this paper's specification of the research technology does make a difference is in the welfare analysis, which is the subject of the next section.

In closing this section, two other issues need to be discussed. The first is the issue of uncertainty. The negative externality in (1) has been based on an intuition drawn from the literature on patent races. In such races multiple researchers pursue the same project; one wins and the others lose. The uncertainty that is intrinsic to this scenario is not reflected in the non-stochastic technology (1). However, it can be shown that under the assumption of no *aggregate* uncertainty, idiosyncratic uncertainty can be introduced into the model so as to reflect the patent race "story." Suppose the uncertainty that a research firm, say, firm k , faces is of the form a_k plus a random variable (or random variables) with expectation zero. Note that output could be greater than or less than a_k . Risk neutral firms will, however, base their decisions on expected output, which is a_k as in (1). Therefore, the firm's profit-maximizing behavior will not be affected. Moreover, the representative agent assumption implies that the single individual of this model owns all shares of all firms. Therefore, even if we allow some firms to succeed and others to fail in patent races, as long as there is no aggregate uncertainty the profit earnings of the representative agent will be unaffected by the idiosyncratic uncertainty faced by the firms. Thus neither the firms nor the representative individual will behave any differently because of uncertainty, and the equilibrium will continue to be exactly as has been described. For more on the use of idiosyncratic uncertainty without aggregate uncertainty see Barro and Sala-i-Martin (1995, page 249 including footnote 7).

Finally, it should be pointed out that the congestion externality focuses our attention on the *quality*, and not just the *quantity*, of research labor. The existing models of endogenous technological progress can incorporate quality differences in research by redefining labor as "effective labor" or "quality adjusted labor." But this approach would capture the contribution of quality only to the extent that quality and quantity are perfectly substitutable. It would miss the role that the quality of research labor plays in reducing the negative effects of the congestion externality.

Smarter researchers will cast their nets wider when fishing for research problems; such researchers can and will abandon "overcrowded" problems for ones that are being investigated by fewer researchers. This tendency will reduce overall congestion levels in research and make its negative externality less burdensome for the economy. If one replaces $f(L_A)$ in the discussion above by $f(L_A, \theta)$, where θ is a labor quality parameter, and if one assumes that $f(L_A, \theta)$ is increasing in θ , then it is readily apparent that both \bar{L}_A and $\bar{g}_A = f(\bar{L}_A, \theta)$ are increasing in θ .

4. Optimality: The Possibility of Overinvestment

In Romer (1990) the amount of human capital employed in the research sector is less than optimal. This is roughly because (i) the market ignores the fact that an idea invented now makes future inventions easier, and (ii) because the monopoly in the intermediate goods sector, itself the result of patents, inhibits the optimal use of the intermediate goods invented by the researchers. This paper's congestion externality introduces a third distortion and raises the possibility of there being *more* investment in R&D than is socially optimal.

The Pareto optimal allocation for the economy described in the last section is the solution to the following social planner's problem: maximize the utility of the representative individual (3), subject to the law of motion for the state variable A (2); to the technological and resource constraints (5), (15), and (8); and to the data of the model, which include the initial condition A_0 , all technological and preference parameters, and the labor endowment \bar{L} .

It is shown in the appendix that if L_A^* solves

$$\frac{1}{(1 - \alpha)f'(L_A)} \rho + L_A = \bar{L}, \quad (25)$$

then $L_{At} = L_A^*$ for all t is the unique solution to the social planner's problem.

The main question of interest in this section is whether the amount of labor employed in the research sector is bigger in the balanced growth equilibrium path or in the balanced growth optimal path. For this purpose, \bar{L}_A , the solution to (21), will have to be compared with L_A^* , the solution to (25).

The assumptions made so far about $f(\cdot)$ are $f(0) = 0$, $f' \geq 0$, and $f'' \leq 0$. These assumptions imply that both $f'(L_A)$ and $f(L_A)/L_A$ are non-increasing in L_A . Thus the left-hand side (LHS) expressions of both (21) and (25) are increasing in L_A . The determination of the equilibrium and optimal levels of L_A is shown in Figure 1. Let the LHS expression of (25) be line B . If the LHS expression of (21) is line A and is, therefore, higher, at $L_A = L_A^*$, than the LHS expression of (25), then \bar{L}_A , the amount of labor allocated to research in equilibrium, would be less than L_A^* . On the other hand, if the LHS expression of (21) is line C and is, therefore, lower, at $L_A = L_A^*$, than the LHS expression of (25), then there would be excessive investment in R&D. It can be shown that for overinvestment to occur, the elasticity of $f(\cdot)$, that is, $L_A f'(L_A)/f(L_A)$, would have to be significantly less than one. And if the elasticity approaches zero, then overinvestment would be ensured.

To take a concrete case, let $f(L_A) = L_A^\beta$. Then, the LHS of (25) minus the LHS of (21) is given by

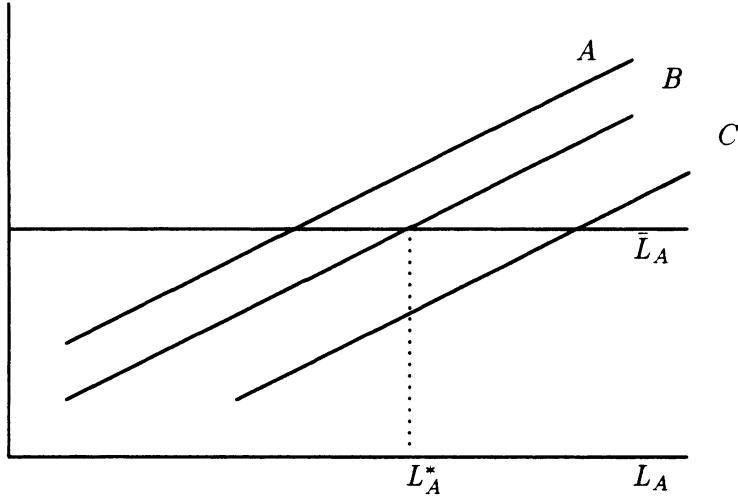


Figure 1.

$$\left(\frac{1}{(1-\alpha)\beta} + 1 - \frac{1}{\alpha(1-\alpha)} \right) \frac{\rho}{L_A^{\beta-1}} + \left(1 - \frac{1}{\alpha(1-\alpha)} \right) L_A .$$

In Romer's linear ($\beta = 1$) case the above expression is negative at all L_A , thus giving Romer's underinvestment result.

Under diminishing returns, however, β is less than one. In particular, as β tends to zero, the above expression not only becomes positive, it tends to infinity at every L_A . Thus it is clear that if diminishing returns are strong enough (i.e., for small enough β) the LHS term of (21) would be less than the LHS term of (25) at $L_A = L_A^*$, and overinvestment would, therefore, obtain in research.

It is important to note that diminishing returns, while necessary for overinvestment to occur, is not sufficient. The congestion externality, which is the motivation for this paper's use of diminishing returns, is only one of several forces that bear upon the differences between \bar{L}_A and L_A^* . There are two distortions, numbered (i) and (ii) in the opening paragraph of this section, that represent tendencies toward underinvestment. So, while the congestion externality does represent a tendency toward overinvestment, it can cause overinvestment only if it is strong enough to outweigh the two other distortions.

Finally, it should be pointed out that the reason why this paper has based itself, not on Romer (1990), but on the Grossman and Helpman (1991)

adaptation of Romer (1990), is because it allows the equilibrium and optimal paths to be worked out for arbitrary initial conditions. In Romer (1990) the balanced growth equilibrium allocation is analyzed for one specific set of initial conditions and the balanced growth optimal allocation is analyzed for another specific set of initial conditions. The two paths are, therefore, not comparable. The model of Grossman and Helpman, on the other hand, has only one state variable where Romer's had two. This does take away some of the richness of the Romer model but allows Grossman and Helpman to formalize the underinvestment result that Romer had sketched.

5. Conclusion

In the current deluge of papers on endogenous technological progress it is generally assumed that the rate of technological progress at any date is linearly increasing in the research effort made at that date. This makes it possible, at least logically, to generate any number of inventions, no matter how large, in any period of time, no matter how short. Since this is grossly unrealistic, it is important to show how linearity can be done away with in models of endogenous growth. This paper has argued that the process by which new economically valuable ideas are generated in contemporary economies is characterized by important "congestion" externalities which yield diminishing returns in the research sector.

The *quality* of labor is shown to be positively related to the growth rate of the model economy in a way that is not captured by the usual "effective labor" approach to modeling labor quality.

The optimality analysis of the model has shown that advocacy of the use of subsidies to encourage investment in research as a correction for market failure may be ignoring other market failures that would suggest a different course of action. The policy debate, therefore, is unlikely to be resolved by mere a priori argument. It is fundamentally an empirical matter.

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Appendix A

The Optimal Allocation

The derivation of (25) will be shown.

The social planner's problem (see Section 4) is to maximize the utility function (3) subject to the consumption good technology (5), the intermediate good technology (8), the research sector technology (2), and the labor constraint (15).

All the constraints are point-in-time or static constraints except for (2). Note also that L_x and L_c do not appear in (2). This implies that the social planner's problem can be broken up into a static allocation problem involving L_x and L_c , and a dynamic allocation problem involving neither L_x nor L_c .

The static allocation problem is to maximize c_t subject to (5), (8) and (15):

$$\max \quad L_{ct}^{1-\alpha} \int_0^{A_t} x_{jt} dj,$$

$$\text{subject to } \int_0^{A_t} x_{jt} dj = L_{xt},$$

$$\text{and } L_{xt} + L_{ct} = \bar{L} - L_{At},$$

where \bar{L} , A_t and L_{At} are taken as given.

The first-order conditions for the static problem can be used to substitute out L_x and L_c from the social planner's problem, which then becomes

$$\max \int_0^{\infty} e^{-\rho t} \ln c_t dt$$

$$\text{subject to } c_t = \alpha^\alpha (1 - \alpha)^{1-\alpha} A_t^{1-\alpha},$$

$$\dot{A}_t = A_t f'(L_{At}),$$

and the initial condition that A_0 is given.

The Hamiltonian for this problem is

$$\mathcal{H} = (1 - \alpha) \ln A_t + \ln(\bar{L} - L_{At}) + \lambda_t A_t f(L_{At}).$$

The optimality conditions are (i) $\partial \mathcal{H} / \partial L_{At} = 0$, (ii) $\dot{\lambda}_t = \rho \lambda_t - \partial \mathcal{H} / \partial A_t$, and the transversality condition (iii) $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda_t A_t = 0$.

Condition (i) gives

$$(\bar{L} - L_{At}) f'(L_{At}) = \frac{1}{\lambda_t A_t}, \quad (26)$$

and condition (ii) and (2) give

$$\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{A}_t}{A_t} = \rho - \frac{1 - \alpha}{\lambda_t A_t}. \quad (27)$$

Defining $M_t \equiv \lambda_t A_t$, one can rewrite (27) as $\dot{M}_t = \rho M_t - (1 - \alpha)$ and condition (iii) as $\lim_{t \rightarrow \infty} e^{-\rho t} M_t = 0$. These two conditions can be shown to imply $\dot{M}_t = 0$ or $M_t = (1 - \alpha)/\rho$, which, when substituted into (26) gives (25).

Zero Profit Equilibrium

Two propositions regarding the research sector's zero profit equilibrium will be proven.

PROPOSITION 1. If $L_A = \bar{L}_A$ solves (21), $f(0) = 0$, $f' \geq 0$, and $f(L_A)/L_A$ is non-increasing in L_A , there is a zero profit equilibrium in the research sector at each t such that $L_{kt} = 1$ for all active firms k and the number of firms is $m_t = L_{At}$. If $f(L_A)/L_A$ is decreasing in L_A this equilibrium is unique.

Proof. From (1), firm k 's profits are $\pi_k = P_{At}\alpha_k - w_t L_k$ or

$$\pi_k = \begin{cases} \left(A \frac{f(L_{-k} + L_k)}{L_{-k} + L_k} P_{At} - w_t \right) L_k, & \text{if } L_k \geq 1 \\ -w_t L_k, & \text{if } 1 > L_k \geq 0. \end{cases} \quad (28)$$

Since \bar{L}_A solves (21) it also satisfies (20). Therefore, if $L_k = 1, k = 1, 2, \dots, m_t$ and $m_t = \bar{L}_A$, then $\pi_k = 0$ for all k . Moreover, if firm k changes L_k unilaterally (i.e., taking L_{-k} as given), π_k will either stay zero or become negative because $f(L_A)/L_A$ is assumed non-increasing. Thus, this is a zero profit equilibrium.

Moreover, if $f(L_A)/L_A$ is decreasing in L_A , a unilateral change in L_k from 1 to any other positive number will clearly reduce π_k from zero to a negative number. Thus $L_k = 1$ must hold for all active firms k in any zero profit equilibrium.

PROPOSITION 2. If $\lim_{L_A \rightarrow \infty} f(L_A)/L_A = 0$ and $f(L_A)/L_A$ is non-increasing, then for any w_t and P_{At} there will exist a zero profit equilibrium such that $L_{kt} = 1$ for all active firms k . Under the additional assumption that $f(L_A)/L_A$ is decreasing in L_A , this equilibrium will be unique.

Proof. Since $\lim_{L_A \rightarrow \infty} f(L_A)/L_A = 0$, for any w_t and P_{At} , either (i) there will exist an \hat{L}_A such that $w_t/P_{At} = A_t f(\hat{L}_A) \hat{L}_A$, or (ii) $w_t/P_{At} > A_t f(L_A)/L_A$ for all L_A . In case (ii) $L_A = 0$ is the zero profit equilibrium because no active firm can have non-negative profits. In case (i) $L_{kt} = 1, k = 1, 2, \dots, m_t$, and $m_t = \hat{L}_{At}$ is a zero profit equilibrium by reasoning similar to that used in Proposition 1. And if $f(L_A)/L_A$ is decreasing in L_A , it is similarly the only such equilibrium.

Appendix B

List of Symbols

- A_t = Number of intermediate goods invented by time t .
- \bar{L} = Labor endowment.

- L_{At} = Labor allocated to research at time t .
- L_{xt} = Labor allocated to the production of intermediate goods at time t .
- L_{ct} = Labor allocated to the production of the consumption good at time t .
- r_t = Real (gross) interest on consumption loans at time t .
- c_t = Consumption at time t .
- ρ = Rate of time preference.
- x_{jt} = Amount of the intermediate good j produced at time t .
- s, t = Indicates time.
- α = Parameter in consumption good technology.
- w_t = Wage rate, in units of the consumption good, at time t .
- j = Indicates an intermediate good.
- P_{jt} = Price, in units of the consumption good, of j at time t .
- π_{jt} = Profits earned by producer of j at time t .
- R_t = Discount factor.
- g_{At} = Growth rate of A_s at time t .
- g_{ct} = Growth rate of c_s at time t .
- θ = Parameter representing entrepreneurial talent.
- λ_t = Shadow price of A_s at time t .
- M_t = Auxilliary variable; $M_t = \lambda_t A_t$.
- \mathcal{H} = Hamiltonian.