Show your work - write down units with all numbers that have them - show all steps.

**Part A - Waves**

Waves have **crests** (highs) and **troughs** (lows). The **wave height** (amplitude) is the difference in height between the crest and the trough. The **wavelength** (L) is the distance between two crests (or troughs). The **period** (T) is the elapsed time between passage of successive wave crests (or troughs). As a wave approaches the shore it slows down from drag on the bottom when water depth is less than half the wavelength (L/2). This L/2 depth is often called the **wave base** for that size wave.

**Wave velocity or celerity** (C)

For any kind of wave, \( C = \frac{L}{T} \) or wave velocity = wavelength divided by period,

which is simply wave speed equals distance divided by time. In deep water the wave velocity can be calculated from either the period (T) or the wavelength (L).

While the size of waves depends on the wind that produces them, the driving force for wave motion, once formed, is gravity \((g, \text{gravitational acceleration})\). Gravity tries to lower the crest, but as the crest is pulled down, it pushes the water in the trough upward. Waves travel at a speed dependent on the wavelength and gravitational acceleration \((g = 9.8 \text{ m/s}^2)\). **In deep water**, \[
C^2 = \frac{gL}{2\pi}
\]

The celerity can be determined from wavelength or period, and wavelength can be determined from the period using the following equations.

\[
C = 1.25 \sqrt{L} \quad C = 1.56T \quad L = 1.56T^2
\]

Note: the constant coefficients (1.25 & 1.56) are derived from solution of the equation above with \(g \& \pi\). The constant coefficients contain hidden units that will cancel with input values and give proper units in result using only units of meters and seconds: wavelength in meters, period in seconds, celerity in meters per second.

**Shallow Water** As waves move from deep water into progressively shallower water, less than L/2 depth, they slow down. The celerity depends on the water depth \((D)\):

\[
C = (9.8 \text{ m/s}^2 \times D)^{1/2} = 3.1 \times D^{1/2}
\]

And as waves slow in shallow water, their wavelength decreases, the waves get steeper and higher. As a wave of a particular height \(h_d\) in deep water \(H_d\) comes into shallow water \(H_s\), its height \(h_s\) is estimated by:

\[
h_s/h_d = (H_d/H_s)^{1/4}
\]

(Australian Bureau of Meteorology; http://www.bom.gov.au/tsunami/info/)
**Part A Questions:**

1) Offshore (deep water) waves may commonly have a period of around 9 seconds. How fast do these waves travel? Calculate and give answer in m/s and also convert to mi/hr. [1 m/s = 2.24 mi/hr].

2) At what depth do the waves in 1 start to feel the bottom as they approach the coastline?

3) Waves in a storm swell may have a period of 18 seconds. How fast do these waves travel in deep water? Calculate and give answer in m/s and also convert to mi/hr.

4) At what depth do the waves in 3 begin to stir the seafloor?

5) Do any of the waves above stir the sea floor of the entire continental shelf (down to about 120 m depth)?

As a wave passes, water molecules move in a circular orbital motion. The orbits grow smaller with depth. The diameter of the orbits at the surface equals the wave height. At increasing depth the orbital diameter is given by:

\[ H_y = H \cdot e^{-2\pi y/L} \]

- \( H_y \): orbital diameter at depth \( y \)
- \( H \): wave height
- \( e \): the constant 2.71828...
- \( y \): depth
- \( L \): wavelength

The velocity of the moving water molecules, \( u \), is the circumference of the orbit (circumference of a circle is \( \pi \) times the diameter) divided by the wave period:

\[ u = \pi \frac{H}{T} \]

6) For the waves in question 1 and 2, with a period of 9 seconds and a wave height of 2 meters, how fast is the water moving (orbital speed) at the surface. Calculate and give answer in both m/s and mi/hr.

7) What is the size of the orbital for water at the \( L/2 \) depth?

8) What is the orbital velocity of water at \( L/2 \)?
Part B - Wave Heights

9) Howell (1972) showed that wind blowing over a large fetch for many hours will produce maximum wave heights proportional to the velocity squared:

\[ H = 0.0023 \, V^2 \]

where \( H \) is wave height in meters and \( V \) is wind velocity in km/hr.

“On February 16, 1982, the world’s largest offshore oil rig was destroyed by a North Atlantic storm. Eighty four oil workers were drowned. Waves 15 m high were reported, as well as 73 km/hr winds which developed freezing spray.” (Rahn, 1996)

a) Use Howell’s equation to estimate the wind speed that would build these 15 m (~49 ft) waves.

b) Use Howell’s equation to estimate the maximum wave size that should be produced by 73 km/hr (~45 mi/hr) winds blowing for many hours.

c) As Howell (1972), numerous other empirical and theoretical studies have addressed the relationship between wind speed, duration, fetch and wave height. The attached figure (Figure 10.10 in Rahn, 1996; slides 3-5 of the Coastal slide presentation) combines the equations into a graphical solution.

Using the figure on the next page, estimate the significant wave height expected to be produced by 73 km/hr winds blowing for 6 hours across a 32 km (20 mi) wide bay (about like Long Island Sound). Give answer in meters and in feet.

10) Tsunamis are waves produced by the sudden offset of the seafloor, for example due to a large earthquake in the ocean crust or a major seafloor slump. These waves have very long wavelengths, so they travel very fast, but they are low (~1m) and unnoticeable to ships as they travel across the deep ocean. It is only when they come into shallow water that they become apparent. The word tsunami is Japanese for harbor wave.

Consider a tsunami generated by a strong earthquake that produced a large offset of the seafloor on the subduction zone off the southern coast of Alaska. The tsunami has a wavelength of 200 km and a wave height of 1 m travelling in water 5,000 m deep in the north Pacific approaching Hawaii.

a) How fast will the tsunami be travelling (in m/s and km/hr)? (hint: is this a deep water or shallow water wave? Compare the wavelength with deep ocean water depth.)

b) When the tsunami comes into water 10 m deep what will the wave velocity be (in m/s and km/hr)?

c) Consider these two wave velocities and describe, in qualitative terms, the effect that this decrease must have on wavelength and wave height (considering how much the waves slowed, how much shorter and higher must the waves grow).

d) Estimate the wave height of this 1 m high tsunami when it moves into 3 m deep water. (hint: use the \( h/h_d \) equation from the first page)
Part C - Littoral Drift on Fire Island

Refer to the portion of the composite Bayshore West and Bayshore East quadrangles on the next page. The Fire Island Lighthouse was built in 1825 to mark the entrance to Fire Island Inlet. Find the lighthouse in the U.S. Coast Guard Reservation near the east end of the section of barrier island shown.

11) Explain why the lighthouse is no longer at the inlet?

12) From what direction must the waves preferentially come?

13) How far is the lighthouse from the inlet? Use the graphical (bar) scale. Measure in feet. Give answer in feet and in miles.

14) What was the average rate of westward accretion (addition) of Fire Island between 1825 and about 1940 when the jetty was built at Democrat Point? Note the end of the jetty is marked by a light.