HOMEWORK - DUE 11/18/2019

MATH 20

(1) Let $f : A \to B$ and $g : B \to C$ be functions. Prove: If f and g are both one-to-one, then $(g \circ f)$ is one-to-one.

(2) In high school, you learn that a function $f : \mathbb{R} \to \mathbb{R}$ is one-to-one if its graph passes the horizontal line test. Explain what this means and why it agrees with our definition of a one-to-one function $f : \mathbb{R} \to \mathbb{R}$.

(3) Consider the function $\sin : \mathbb{R} \to \mathbb{R}$. Restrict the domain and target so that the function is a bijection. Use this to determine the domain and range of the inverse trigonometric function arcsin. Do the same for the trigonometric functions cosine and tangent.

(4) Consider the function $f(x) = \dfrac{3x-6}{2x+1}$.
   (a) Explicitly find a function $g(y)$ that is inverse to f. You will do this by solving for x in the equation $y = \dfrac{3x-6}{2x+1}$.
   (b) The functions f and g are inverse functions on what sets? (i.e. What is domain and range of $f(x)$ and $g(y)$?)
   (c) Show explicitly (algebraically) that $(g \circ f)(x) = x$ and $(f \circ g)(y) = y$ for all $x \in \text{Domain}(f)$ and $y \in \text{Domain}(g)$.

(5) Consider the function

$$f(x) = (x^2 - 2x + 3)e^x.$$

   (a) Try to find an inverse to f^{-1} explicitly, using algebra. (You will probably be unsuccessful.)
   (b) Use calculus to show that $f : \mathbb{R} \to (0, \infty)$ is a bijection.

(6) Suppose that $f : A \to B$ and $g : B \to C$ are invertible functions. Show that $(g \circ f) : A \to C$ is invertible, and that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.
(Hint: The proof is simple; it follows very directly from the definition of inverse function.)

Date: November 6, 2019.