MATH 521 HOMEWORK G
DUE 11/14/16

(1) Taken from "The Compleat Strategyst" by John Williams.

"I know a good game," says Alex. "We point fingers at each other; either one finger or two fingers. If we match with one finger, you buy me one Daiquiri, If we match with two fingers, you buy me two Daiquiris. If we dont match I let you off with a payment of a dime. It'll help pass the time."

Olaf appears quite unmoved. "That sounds like a very dull game at least in its early stages." His eyes glaze on the ceiling for a moment and his lips flutter briefly; he returns to the conversation with: "Now if youd care to pay me 42 cents before each game, as a partial compensation for all those 55-cent drinks I'll have to buy you, then I'd be happy to pass the time with you."

Analyze the above game with and without the side payment. Determine optimal strategies and the value of the game to Olaf. What side payment would make the game fair?

(2) Solve the following IP graphically.

$$\begin{aligned} \text{Maximize} \quad & z = 3x + 4y \\ \text{subject to} \quad & x + 3y \leq 10 \\ & 3x + 2y \leq 13 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{aligned}$$

(3) Consider the logical operator AND defined by

$$(0 \,\text{AND}\, 0) = 0, \quad (0 \,\text{AND}\, 1) = 0, \quad (1 \,\text{AND}\, 0) = 0, \quad (1 \,\text{AND}\, 1) = 1.$$

Show that, for $x_1, x_2 \in \{0, 1\}$, we can define $w = (x_1 \,\text{AND}\, x_2)$ by

$$2w \leq x_1 + x_2 \leq w + 1, \quad w \in \{0, 1\}.$$

Date: November 7, 2016.