

- c) Why does the Vogel approximation method perform so well, compared to other methods of finding an initial feasible solution?
10. Colonel Cutlass, having just taken command of the brigade, has decided to assign men to his staff based on previous experience. His list of major staff positions to be filled is adjutant (personnel officer), intelligence officer, operations officer, supply officer, and training officer. He has five men he feels could occupy these five positions. Below are their years of experience in the several fields.

	<i>Adjutant</i>	<i>Intelligence</i>	<i>Operations</i>	<i>Supply</i>	<i>Training</i>
Major Muddle	3	5	6	2	2
Major Whiteside	2	3	5	3	2
Captain Kid	3	—	4	2	2
Captain Klutch	3	—	3	2	2
Lt. Whiz	—	3	—	1	—

Who, based on experience, should be placed in which positions to give the greatest total years of experience for all jobs? (*Hint*. A basis, even if degenerate, is a spanning tree.)

11. Consider the following linear program:

$$\text{Minimize } z = 3x_{12} + 2x_{13} + 5x_{14} + 2x_{41} + x_{23} + 2x_{24} + 6x_{42} + 4x_{34} + 4x_{43},$$

subject to:

$$\begin{aligned} x_{12} + x_{13} + x_{14} - x_{41} &\leq 8, \\ x_{12} - x_{23} - x_{24} + x_{42} &\geq 4, \\ x_{34} - x_{13} - x_{23} - x_{43} &\leq 4, \\ x_{14} + x_{34} + x_{24} - x_{42} - x_{43} &\geq 5, \\ \text{all } x_{ij} &\geq 0. \end{aligned}$$

- a) Show that this is a network problem, stating it in general minimum-cost flow form. Draw the associated network and give an interpretation to the flow in this network.
- b) Find an initial feasible solution. (*Hint*. Exploit the triangular property of the basis.)
- c) Show that your initial solution is a spanning tree.
- d) Solve completely.
12. A lot of three identical items is to be sequenced through three machines. Each item must be processed first on machine 1, then on machine 2, and finally on machine 3. It takes 20 minutes to process one item on machine 1, 12 minutes on machine 2, and 25 minutes on machine 3. The objective is to minimize the total work span to complete all the items.
- a) Write a linear program to achieve our objective. (*Hint*. Let x_{ij} be the starting time of processing item i on machine j . Two items may not occupy the same machine at the same time; also, an item may be processed on machine $(j + 1)$ only after it has been completed on machine j .)
- b) Cast the model above as a network problem. Draw the associated network and give an interpretation in terms of flow in networks. (*Hint*. Formulate and interpret the dual problem of the linear program obtained in (a).)
- c) Find an initial feasible solution; solve completely.
13. A manufacturer of small electronic calculators is working on setting up his production plans for the next six months. One product is particularly puzzling to him. The orders on hand for the coming season are:

<i>Month</i>	<i>Orders</i>
January	100
February	150
March	200
April	100
May	200
June	150