

**MATH 521 HOMEWORK A**  
**DUE 9/17/18**

- (1) Problems 11, 17, 19, 25, 27 from Berresford–Rockett (see following pages).
- (2) Write examples LPs that have:
  - (a) No feasible solution
  - (b) A feasible solution but no optimal solution
  - (c) Infinitely many optimal solutions
- (3) Read Chapter 1 (pp. 1–10) and the first part of Chapter 2, up through and including 2.3 (pp. 11–19).

Problems 4, 5 refer to the following: In the textbook, they say an LP is a problem of the form  
Maximize  $\mathbf{c}^T \mathbf{x}$  subject to the  
constraints  $A\mathbf{x} \leq \mathbf{b}$ .

- (4) Consider problem 17 from Berresford–Rockett. Rewrite the LP in the above form.
- (5) Suppose you have the following as a constraint in an optimization problem, where  $f(x_1, \dots, x_n)$  is a linear function on  $n$  variables:

$$|f(x_1, \dots, x_n)| \leq b.$$

Strictly speaking, this constraint is not linear. Show that you can rewrite it in terms of linear constraints so that the feasible solutions, and hence the optimal solution, remain unchanged.

- (6) Next week, either bring a computer with Microsoft Excel to class (Google Sheets is actually okay, but Excel preferable) OR email me in advance to tell me that you won't be able to bring one.

Solve each linear programming problem by sketching the region and labeling the vertices, deciding whether a solution exists, and then finding it if it does exist.

11. Maximize  $P = 30x + 40y$   
 Subject to  $\begin{cases} 2x + y \leq 16 \\ x + y \leq 10 \\ x \geq 0, y \geq 0 \end{cases}$
12. Maximize  $P = 80x + 70y$   
 Subject to  $\begin{cases} x + 2y \leq 18 \\ x + y \leq 10 \\ x \geq 0, y \geq 0 \end{cases}$
13. Minimize  $C = 15x + 45y$   
 Subject to  $\begin{cases} 2x + 5y \geq 20 \\ x \geq 0, y \geq 0 \end{cases}$
14. Minimize  $C = 35x + 25y$   
 Subject to  $\begin{cases} 5x + 3y \geq 60 \\ x \geq 0, y \geq 0 \end{cases}$
15. Maximize  $P = 4x + 5y$   
 Subject to  $\begin{cases} 2x + y \leq 50 \\ x + 3y \leq 75 \\ x \geq 0, y \geq 0 \end{cases}$
16. Maximize  $P = 7x + 8y$   
 Subject to  $\begin{cases} 3x + y \leq 90 \\ x + 2y \leq 60 \\ x \geq 0, y \geq 0 \end{cases}$
17. Minimize  $C = 12x + 10y$   
 Subject to  $\begin{cases} 4x + y \geq 40 \\ 2x + 3y \geq 60 \\ x \geq 0, y \geq 0 \end{cases}$
18. Minimize  $C = 20x + 30y$   
 Subject to  $\begin{cases} 3x + 2y \geq 120 \\ x + 4y \geq 80 \\ x \geq 0, y \geq 0 \end{cases}$
19. Maximize  $P = 5x + 3y$   
 Subject to  $\begin{cases} 2x + y \leq 90 \\ x + y \leq 50 \\ x + 2y \leq 90 \\ x \geq 0, y \geq 0 \end{cases}$
20. Maximize  $P = 6x + 5y$   
 Subject to  $\begin{cases} x + 2y \leq 96 \\ x + y \leq 54 \\ 2x + y \leq 96 \\ x \geq 0, y \geq 0 \end{cases}$
21. Maximize  $P = 10x + 12y$   
 Subject to  $\begin{cases} 3x + 2y \leq 180 \\ 4x + y \leq 120 \\ 3x + y \leq 105 \\ x \geq 0, y \geq 0 \end{cases}$
22. Maximize  $P = 20x + 15y$   
 Subject to  $\begin{cases} 2x + 3y \leq 60 \\ x + 4y \leq 40 \\ x + 3y \leq 33 \\ x \geq 0, y \geq 0 \end{cases}$
23. Minimize  $C = 20x + 25y$   
 Subject to  $\begin{cases} 3x + y \geq 60 \\ x + y \geq 42 \\ x + 3y \geq 60 \\ x \geq 0, y \geq 0 \end{cases}$
24. Minimize  $C = 50x + 35y$   
 Subject to  $\begin{cases} x + 3y \geq 72 \\ x + y \geq 48 \\ 3x + y \geq 72 \\ x \geq 0, y \geq 0 \end{cases}$

## APPLIED EXERCISES

Formulate each situation as a linear programming problem by identifying the variables, the objective function, and the constraints. Be sure to state clearly the meaning of each variable. Determine whether a solution exists, and if it does, find it. State your final answer in terms of the original question.

- 25. BUSINESS: Livestock Management** A rancher raises goats and llamas on his 400-acre ranch. Each goat needs 2 acres of land and requires \$100 of veterinary care per year, while each llama needs 5 acres of land and requires \$80 of veterinary care per year. The rancher can afford no more than \$13,200 for veterinary care this year. If the expected profit is \$60 for each goat and \$90 for each llama, how many of each animal should he raise to obtain the greatest possible profit?
- 26. BUSINESS: Agriculture Management** A farmer grows wheat and barley on her 500-acre farm. Each acre of wheat requires 3 days of labor to plant, tend, and harvest, while each acre of barley requires 2 days of labor. The farmer and her hired field hands can provide no more than 1200 days of labor this year. If the expected profit is \$50 for each acre of wheat and \$40 for each acre of barley, how many acres of each crop should she grow to obtain the greatest possible profit?
- 27. BUSINESS: Resource Allocation** A sailboat company manufactures fiberglass prams and yawls. The amount of molding, painting, and finishing needed for each is shown in the table, together with the number of hours of skilled labor available for each task. If the expected profit is \$150 for each pram and \$180 for each yawl, how many of each kind of sailboat should the company manufacture to obtain the greatest possible profit?

	Pram	Yawl	Labor Available
Molding	3 hours	6 hours	150 hours
Painting	3 hours	2 hours	114 hours
Finishing	2 hours	6 hours	132 hours

- 28. BUSINESS: Production Planning** A small jewelry company prepares and mounts semi-precious stones. There are 10 lapidaries (who cut and polish the stones) and 12 jewelers (who mount the stones in gold settings). Each employee works 7 hours each day. Each tray of agates requires 5 hours of cutting and polishing and 4 hours of mounting, while each tray of onyxes requires 2 hours of cutting and polishing and 3 hours of mounting. If the profit is \$15 for each tray of agates and \$10 for each tray of onyxes, how many trays of each stone should be processed each day to obtain the greatest possible profit?
- 29. ENVIRONMENTAL SCIENCE: Waste Management** The Marshall County trash incinerator in Norton burns 10 tons of trash per hour and co-generates 6 kilowatts of electricity, while the Wiseburg incinerator burns 5 tons per hour and co-generates 4 kilowatts. The county needs to burn at least 70 tons of trash and co-generate at least 48 kilowatts of electricity every day. If the Norton incinerator costs \$80 per hour to operate and the Wiseburg incinerator costs \$50, how many hours should each incinerator operate each day with the least cost to the county?
- 30. GENERAL: Disaster Relief** An international relief agency has been asked to provide medical support to a Caribbean island devastated by a recent hurricane. The agency estimates that it must be able to perform at least 50 major surgeries, 78 minor surgeries, and 130 outpatient services each day. The daily capacities of its portable field hospitals and clinics are given in the table, along with the daily operating costs. How many field hospitals and clinics should the agency airlift to the island to provide the needed help at the least daily cost?

	Major Surgeries	Minor Surgeries	Outpatient Services	Daily Cost
Field hospitals	5	3	2	\$2000
Clinics	0	2	10	\$500