MATH 521 HOMEWORK J
DUE 11/26/18

(1) Solve the following IP graphically.

$$\begin{aligned} \text{Maximize} \quad & z = 3x + 4y \\ \text{subject to} \quad & x + 3y \leq 10 \\ & 3x + 2y \leq 13 \\ & x_1, x_2 \geq 0 \\ & x_1, x_2 \in \mathbb{Z} \end{aligned}$$

(2) Solve the following IP (by hand, with help from calculator/computer if you wish) using the branch and bound algorithm.

$$\begin{aligned} \text{Maximize} \quad & z = 40x_1 + 8x_2 + 15x_3 + x_4 \\ \text{subject to} \quad & 8x_1 + 4x_2 + 5x_3 + x_4 \leq 10 \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

(3) Exercise 1 on next page. You do not need to solve the IP, only to set it up.

(4) Consider the logical operator AND defined by

$$(0 \,\text{AND}\, 0) = 0, \quad (0 \,\text{AND}\, 1) = 0, \quad (1 \,\text{AND}\, 0) = 0, \quad (1 \,\text{AND}\, 1) = 1.$$

Show that, for $x_1, x_2 \in \{0, 1\}$, we can define $w = (x_1 \,\text{AND}\, x_2)$ by

$$2w \leq x_1 + x_2 \leq w + 1, \quad w \in \{0, 1\}.$$

Date: November 19, 2018.

EXERCISES

1. As the leader of an oil-exploration drilling venture, you must determine the least-cost selection of 5 out of 10 possible sites. Label the sites $S_1, S_2, \ldots, S_{10}$, and the exploration costs associated with each as $C_1, C_2, \ldots, C_{10}$.

   Regional development restrictions are such that:

   i) Evaluating sites S_1 *and* S_7 will prevent you from exploring site S_8.
   ii) Evaluating site S_3 *or* S_4 prevents you from assessing site S_5.
   iii) Of the group S_5, S_6, S_7, S_8, only two sites may be assessed.

   Formulate an integer program to determine the minimum-cost exploration scheme that satisfies these restrictions.

2. A company wishes to put together an academic ''package'' for an executive training program. There are five area colleges, each offering courses in the six fields that the program is designed to touch upon.

   The package consists of 10 courses; each of the six fields must be covered.

   The tuition (basic charge), assessed when at least one course is taken, at college i is T_i (independent of the number of courses taken). Moreover, each college imposes an additional charge (covering course materials, instructional aids, and so forth) for *each* course, the charge depending on the college and the field of instructions.

   Formulate an integer program that will provide the company with the minimum amount it must spend to meet the requirements of the program.

3. The marketing group of A. J. Pitt Company is considering the options available for its next advertising campaign program. After a great deal of work, the group has identified a selected number of options with the characteristics shown in the accompanying table.

	TV	*Trade magazine*	*Newspaper*	*Radio*	*Popular magazine*	*Promotional campaign*	*Total resource available*
Customers reached	1,000,000	200,000	300,000	400,000	450,000	450,000	–
Cost ($)	500,000	150,000	300,000	250,000	250,000	100,000	1,800,000
Designers needed (man-hours)	700	250	200	200	300	400	1,500
Salesmen needed (man-hours)	200	100	100	100	100	1,000	1,200

   The objective of the advertising program is to maximize the number of customers reached, subject to the limitation of resources (money, designers, and salesman) given in the table above. In addition, the following constraints have to be met:

   i) If the promotional campaign is undertaken, it needs either a radio or a popular magazine campaign effort to support it.
   ii) The firm cannot advertise in both the trade and popular magazines.

   Formulate an integer-programming model that will assist the company to select an appropriate advertising campaign strategy.

4. Three different items are to be routed through three machines. Each item must be processed first on machine 1, then on machine 2, and finally on machine 3. The sequence of items may differ for each machine. Assume that the times t_{ij} required to perform the work on item i by machine j are known and are integers. Our objective is to minimize the total time necessary to process all the items.

   a) Formulate the problem as an integer programming problem. [*Hint*. Let x_{ij} be the starting time of processing item i on machine j. Your model must prevent two items from occupying the same machine at the same time; also, an item may not start processing on machine $(j + 1)$ unless it has completed processing on machine j.]