Math 521 Midterm Assignment

Due by Tues Nov 20th

Please write out answers to problems, and also email me your Excel File.

- P is the number of production days in one week,
- s_j is the storage space required by *one* item of type j,
- S is the total storage space available for the week's production,
- r_j is the unit profit for each product of type j,
- d_j is the weekly maximum demand for an item of type j.
- b_j is the weekly minimum demand for an item of type j.

Problem 2

(10 points total). Problem 2 reviews the transformations from nonlinear constraints or objectives into linear constraints and objectives, as mentioned in the second lecture and discussed in the tutorial "LP Transformation Tricks".

Identify which of the following optimization problems can be reformulated as a single Linear Program, and write the corresponding equivalent LP formulation. For those problems that cannot be reformulated, the answer should identify all the troublesome constraint(s) and/or objective.

(a) (5 points) Problem formulation:

$$\left.\begin{array}{rlcl} \min & \max\{2.3x_1 + x_2, 4.3x_1 - 0.5x_2, 2.5x_1 + 3.5x_2\} & & \\ \text{s.t.:} & & & \\ \text{Constr1}: & x_1/(x_1+x_2) & \leq & 0.5 \\ \text{Constr2}: & 10x_1 + 28x_2 & = & 3.4 \\ \text{Constr3}: & x_1 + x_2 & \geq & 0 \\ & x_1 & \text{free} & \\ & x_2 & \text{free} & \end{array}\right\} \quad (1)$$

(b) (5 points) Problem formulation:

$$\left.\begin{array}{rlcl} \min & |0.8x_1 + 0.9x_2| & & \\ \text{s.t.:} & & & \\ \text{Constr1}: & |0.9x_1 + 1.2x_2| & \leq & 10 \\ & x_1 & \geq & 0 \\ & x_2 & & \text{free} \end{array}\right\} \quad (2)$$

Problem 3 (First group of students)

(55 points total) This problem is based on Problem 11 of Applied Mathematical Programming, Chapter 1. This is a more complex model than the one in Problem 1. For many product mix examples, each product uses a fixed set of materials. In this example, if one makes one gallon Deluxe, one does not know a priori how much of Additive A and Additive C are used. In fact, the decision maker must determine the optimal amounts of A and C to be added to Deluxe.

A corporation that produces gasoline and oil specialty additives purchases four grades of petroleum distillates, A, B, C, and D. The company then combines the four according to specifications of the maximum or minimum percentages of grades A, C, or D in each blend, given in Table 1.

Mixture	Max % allowed for Additive A	Min % allowed for Additive C	Max % allowed for Additive D	Selling price $/gallon
Deluxe	60%	20%	10%	7.9
Standard	15%	60%	25%	6.9
Economy	–	50%	45%	5.0

Table 1: Specifications of the three mixtures.

Supplies of the three basic additives and their costs are given in Table 2.

Distillate	Max quantity available per day (gals)	Cost $/gallon
A	4000	0.60
B	5000	0.52
C	3500	0.48
D	5500	0.35

Table 2: Supplies and costs of petroleum grades.

(a) (15 points) Formulate a Linear Program to determine the production policy that maximizes profits.

(b) (10 points) EXCEL SUBMISSION Write a spreadsheet for the problem and solve the problem using Excel Solver, following the guidelines given in the Excel Workbook that comes with this problem set. (Hint: The optimal value is \$102986.7 . In the optimal solution, there are 12500 gallons of mixture Deluxe and 1666.667 gallons of mixture Standard. No mixture Economy is produced.)

(c) Use the Excel spreadsheet to answer the following questions:

i) (4 points) No mixture Economy is produced in the optimal solution. What would the minimum selling price need to be in order for Economy to be worth producing? (Be accurate to within 5 cents).

ii) (4 points) There are constraints on the maximum amount of distillate D that is used in producing the different mixtures. If you could violate any of these constraints on distillate D, would you? Which constraint would be most advantageous to delete?

iii) (4 points) Suppose that you can increase the selling price of Deluxe. What would be the increase in the profit if the selling price of mixture Deluxe per gallon increases to p for $p =$ \$ 7.95, \$8.00, and \$8.05 . (Assume that the selling price of mixtures Standard and Economy remain \$6.9 and \$5.0 per gallon, respectively.) The increase is the difference between the new profit and the profit from Part (b).

iv) (4 points) Based on your answer in Part (iv), what do you think will be the profit if the selling price of mixture Deluxe increases by 5%? (Verify that you are correct.) What is the formula for the optimum profit if the selling price of Deluxe per gallon increased by p? (You may assume that p is between 1% and 10%).

v) (4 points) Based on your formula in part (iii), what is the contribution if the selling price of Deluxe per gallon increases to 8.5. Use Excel solver to see if the formula is correct. (It wont be.) Use Excel solver to determine the maximum value of p for which your formula is correct. (Be accurate to within 5 cents).

vi) (5 points) The quantity available of distillate A is 4000 gals per day which is entirely used up in an optimal mixture. If you could buy (just a little bit) more, would you do so? At what price would be worthwhile to buy more?

(d) (10 points) Write an algebraic formulation for the problem using the following notation:

- m is the number of mixtures (final products),
- n is the number of basic petroleum grades (additives),
- p_j is the selling price of mixture j,
- c_i is the cost of additive i,
- a_i is the maximum availability of additive i,
- r_{ij} is the minimum percentage of additive i that is required in mixture j,
- q_{ij} is the maximum percentage of additive i that can be blended in for mixture j,

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