Elementary Row Operations

1. Interchange two rows ($R_i \leftrightarrow R_j$)
2. Multiply one row by a nonzero number ($cR_i \to R_i$)
3. Add a multiple of one row to a different row ($cR_i + R_j \to R_j$)

Elementary row operations do not change the solution set. Thus we can solve a system of linear equations by the following *Gauss-Jordan Elimination procedure*:

1. Write the system as an augmented matrix.
2. Apply elementary row operations to transform the coefficient matrix into *reduced row echelon form*. Or, use a computer/calculator!
    (a) Interchange rows, if necessary, to make the top left entry ("first pivot") non-zero.
    (b) Divide Row 1 by the pivot to get "leading 1."
    (c) Subtract multiples of Row 1 to achieve 0s beneath the first pivot.
    (d) Interchange Row 2 with a later rows, if necessary, to make the the second entry of Row 2 ("second pivot") non-zero.
    (e) Divide Row 2 by its pivot to get leading 1 in row 2.
    (f) Subtract multiples of Row 2 to achieve 0s beneath the second pivot, etc.
    (g) Continue this process until you run out of columns. At this point your matrix will be in *row echelon form*.
    (h) Starting at the right-most leading 1, subtract multiples of that row to eliminate all entries in the column above the leading 1.
    (i) Move to left to the next leading 1, perform the same procedure.
    (j) Continue this process until you reach the first column.
3. Determine whether there is no solution, one solution, or infinitely many solutions. If there are infinitely many solutions, parameterize them by introducing a *free variable* for each column without a leading 1.