Math 615
Week 1

Elementary Row Operations

1. Interchange two rows ($R_i \leftrightarrow R_j$)
2. Multiply one row by a nonzero number ($cR_i \to R_i$)
3. Add a multiple of one row to a different row ($cR_i + R_j \to R_j$)

Elementary row operations do not change the solution set. Thus we can solve a system of linear equations by the following *Gauss-Jordan Elimination procedure*:

1. Write the system as an augmented matrix.
2. Apply elementary row operations to transform the coefficient matrix into *reduced row echelon form.* Or, use a computer/calculator!
   (a) Interchange rows, if necessary, to make the top left entry ("first pivot") non-zero.
   (b) Divide Row 1 by the pivot to get "leading 1."
   (c) Subtract multiples of Row 1 to achieve 0s beneath the first pivot.
   (d) Interchange Row 2 with a later rows, if necessary, to make the the second entry of Row 2 ("second pivot") non-zero.
   (e) Divide Row 2 by its pivot to get leading 1 in row 2.
   (f) Subtract multiples of Row 2 to achieve 0s beneath the second pivot, etc.
   (g) Continue this process until you run out of columns. At this point your matrix will be in *row echelon form.*
   (h) Starting at the right-most leading 1, subtract multiples of that row to eliminate all entries in the column above the leading 1.
   (i) Move to left to the next leading 1, perform the same procedure.
   (j) Continue this process until you reach the first column.
3. Determine whether there is no solution, one solution, or infinitely many solutions. If there are infinitely many solutions, parameterize them by introducing a *free variable* for each column without a leading 1.

Sample Problems

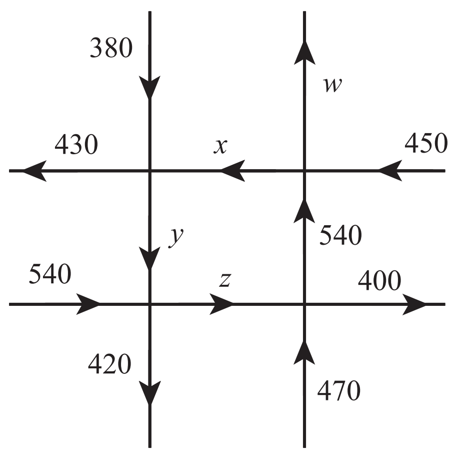
Each of these problems can be solved by writing down a system of linear equations and applying Gaussian Elimination.

1. Butane (C_4H_{10}) burns in the presence of oxygen to form carbon dioxide and water. The chemical equation has the form

$$a\,C_4H_{10} + b\,O_2 \longrightarrow c\,CO_2 + d\,H_2O.$$

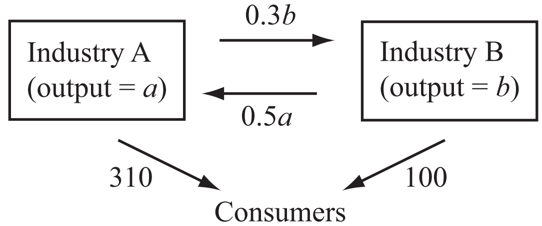
   Find the smallest whole-number values of a, b, c and d that balance the equation. *Write a linear system for the unknowns and solve as done in class.*

2. The number of vehicles flowing along various portions of four one-way streets was measured for 1 hour; the results are shown in the diagram. Find the unknown flows w, x, y, z by setting up and solving a linear system.

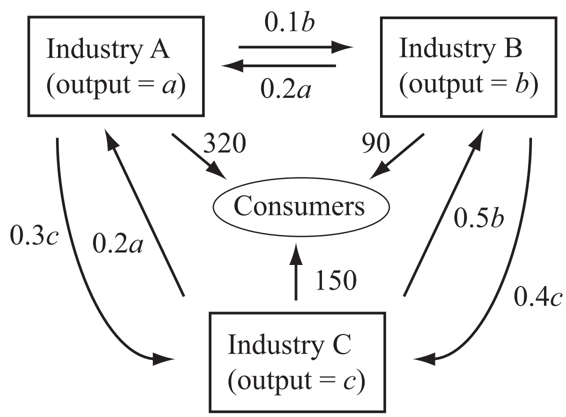


3. The diagram below shows the flow of the outputs between two interdependent industries and the consumer demand for their products (in millions of dollars per year). For example, the arrow labeled "$0.3b$" indicates that Industry B requires $0.3b$ dollars of the Industry A's output as ingredients to produce b dollars of their output.

   Find the outputs a and b needed to satisfy both consumer and inter-industry demand. Assume that everything an industry produces is outputted to consumers or another industry. *Begin by writing, for each industry, the linear equation that says: amount produced = amount consumed.*



4. Try the same with three industries, related according to the following diagram. *This isn't as complicated as it looks!*



5. There is a "quick method" to solve linear systems, described below. Use this method to write down (in our standard form) the solutions to the following linear systems.

(a)
$$\begin{aligned} x + 2y + z &= 5 \\ 2x - 3y + 5z &= -17 \\ 4x + 8y - z &= 30 \end{aligned}$$

(b)
$$\begin{aligned} x_1 + x_2 - 5x_3 + 3x_4 - 4x_5 &= 9 \\ 2x_1 - x_2 + 6x_3 - 3x_4 + x_5 &= 18 \\ -x_1 + 2x_2 + 4x_3 - 4x_4 + 6x_5 &= 19 \end{aligned}$$

Quick Method: Go to WolframAlpha.com. Type in the equations, separated by commas. Press Return.

For (b), give the variables simpler names before entering. Write the answer in both "exact form" and "approximate form".

Homework: (assigned 9/9/19, due 9/16/19)
From textbook: p. 10: 1.19, 1.22, 1.37.
Problems 1, ~~4~~ above. (If you have difficulty with 4, do 3 first as a warmup)