MTH 615 Quiz 2 10/21/19

1. Show that $4+2x \in \text{Span}(1+x, 1-x)$

$$r_1(1+x) + r_2(1-x) = 4+2x$$

$$(r_1+r_2)1 + (r_1-r_2)x = 4(1) + 2\cdot x$$

$$\begin{cases} r_1 + r_2 = 4 \\ r_1 - r_2 = 2 \end{cases} \Rightarrow \begin{cases} r_1 = 3 \\ r_2 = 1 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array}\right] \to \begin{bmatrix} 1 & 1 & 4 \\ 0 & -2 & -2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\boxed{3(1+x) + 1(1-x) = 4+2x}$$

2. Show that $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \notin \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)$

Solve $r_1 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + r_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (*)

$$\left[\begin{array}{cc|c} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}\right] \rightsquigarrow \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

No Solution

There is no solution to (*), hence $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \notin \text{Span}\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right).$