Math 615: October 14, 2015
The Table Function

Question: Are the functions {sin(x), sin(2x), sin(3x)} linearly independent in the vector space of real-valued functions?

To show they are linearly independent, we must show that the only solution (r1, r2, r3) to the equation r1 sin(x) + r2 sin(2x) + r3 sin(3x) = 0 is r1=r2=r3=0. Note that this equation is an equation of functions; it must hold for all values of x.

Therefore, we can try to evaluate at a few values of x to obtain a linear system. Evaluating at x=1, x=2 and x=3 gives us the following, we see that the three functions are linearly independent.

```
RowReduce[( Sin[1]  Sin[2]  Sin[3]
            Sin[2]  Sin[4]  Sin[6]
            Sin[3]  Sin[6]  Sin[9] )] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Can we “automate” this process? Yes! Use the Table function. Here is the documentation.

```
? Table
```

Table[*expr*, *n*] generates a list of *n* copies of *expr*.

Table[*expr*, {*i*, i_{max}}] generates a list of the values of *expr* when *i* runs from 1 to i_{max}.

Table[*expr*, {*i*, i_{min}, i_{max}}] starts with $i = i_{min}$.

Table[*expr*, {*i*, i_{min}, i_{max}, *di*}] uses steps *di*.

Table[*expr*, {*i*, {i_1, i_2, ...}}] uses the successive values i_1, i_2, ...

Table[*expr*, {*i*, i_{min}, i_{max}}, {*j*, j_{min}, j_{max}}, ...] gives a nested list. The list associated with *i* is outermost. >>

In the above example, we can perform our calculation by first using the Table to create our matrix.

```
(a = Table[ {Sin[x], Sin[2 x], Sin[3 x]}, {x, 1, 3}] ) // MatrixForm
RowReduce[a] // MatrixForm
```

$$\begin{pmatrix} \text{Sin}[1] & \text{Sin}[2] & \text{Sin}[3] \\ \text{Sin}[2] & \text{Sin}[4] & \text{Sin}[6] \\ \text{Sin}[3] & \text{Sin}[6] & \text{Sin}[9] \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In fact, we could use the Table function *twice* to create our matrix. First note that...

```
Table[Sin[k x], {k, 1, 3}]
```

```
{Sin[x], Sin[2 x], Sin[3 x]}
```

Therefore, we can use nested Table functions to create a list of lists.

```
a = Table[Table[Sin[k x], {k, 1, 3}], {x, 1, 3}]
MatrixForm[a]
RowReduce[a] // MatrixForm
```

```
{{Sin[1], Sin[2], Sin[3]}, {Sin[2], Sin[4], Sin[6]}, {Sin[3], Sin[6], Sin[9]}}
```

$$\begin{pmatrix} \text{Sin}[1] & \text{Sin}[2] & \text{Sin}[3] \\ \text{Sin}[2] & \text{Sin}[4] & \text{Sin}[6] \\ \text{Sin}[3] & \text{Sin}[6] & \text{Sin}[9] \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Question: Is {sin(x), sin(2x), sin(3x), ..., sin(25x) } linearly independent?

Solution: Use the same process as above. Note that we will need to evaluate the functions at at least 25 different values of x.

```
a = Table[Table[Sin[k x], {k, 1, 25}], {x, 1, 25}];
MatrixForm[a]
RowReduce[a] // MatrixForm
```

Sin[1]	Sin[2]	Sin[3]	Sin[4]	Sin[5]	Sin[6]	Sin[7]	Sin[8]	Sin[9]	S
Sin[2]	Sin[4]	Sin[6]	Sin[8]	Sin[10]	Sin[12]	Sin[14]	Sin[16]	Sin[18]	S
Sin[3]	Sin[6]	Sin[9]	Sin[12]	Sin[15]	Sin[18]	Sin[21]	Sin[24]	Sin[27]	S
Sin[4]	Sin[8]	Sin[12]	Sin[16]	Sin[20]	Sin[24]	Sin[28]	Sin[32]	Sin[36]	S
Sin[5]	Sin[10]	Sin[15]	Sin[20]	Sin[25]	Sin[30]	Sin[35]	Sin[40]	Sin[45]	S
Sin[6]	Sin[12]	Sin[18]	Sin[24]	Sin[30]	Sin[36]	Sin[42]	Sin[48]	Sin[54]	S
Sin[7]	Sin[14]	Sin[21]	Sin[28]	Sin[35]	Sin[42]	Sin[49]	Sin[56]	Sin[63]	S
Sin[8]	Sin[16]	Sin[24]	Sin[32]	Sin[40]	Sin[48]	Sin[56]	Sin[64]	Sin[72]	S
Sin[9]	Sin[18]	Sin[27]	Sin[36]	Sin[45]	Sin[54]	Sin[63]	Sin[72]	Sin[81]	S
Sin[10]	Sin[20]	Sin[30]	Sin[40]	Sin[50]	Sin[60]	Sin[70]	Sin[80]	Sin[90]	Si
Sin[11]	Sin[22]	Sin[33]	Sin[44]	Sin[55]	Sin[66]	Sin[77]	Sin[88]	Sin[99]	Si
Sin[12]	Sin[24]	Sin[36]	Sin[48]	Sin[60]	Sin[72]	Sin[84]	Sin[96]	Sin[108]	Si
Sin[13]	Sin[26]	Sin[39]	Sin[52]	Sin[65]	Sin[78]	Sin[91]	Sin[104]	Sin[117]	Si
Sin[14]	Sin[28]	Sin[42]	Sin[56]	Sin[70]	Sin[84]	Sin[98]	Sin[112]	Sin[126]	Si
Sin[15]	Sin[30]	Sin[45]	Sin[60]	Sin[75]	Sin[90]	Sin[105]	Sin[120]	Sin[135]	Si
Sin[16]	Sin[32]	Sin[48]	Sin[64]	Sin[80]	Sin[96]	Sin[112]	Sin[128]	Sin[144]	Si
Sin[17]	Sin[34]	Sin[51]	Sin[68]	Sin[85]	Sin[102]	Sin[119]	Sin[136]	Sin[153]	Si
Sin[18]	Sin[36]	Sin[54]	Sin[72]	Sin[90]	Sin[108]	Sin[126]	Sin[144]	Sin[162]	Si
Sin[19]	Sin[38]	Sin[57]	Sin[76]	Sin[95]	Sin[114]	Sin[133]	Sin[152]	Sin[171]	Si
Sin[20]	Sin[40]	Sin[60]	Sin[80]	Sin[100]	Sin[120]	Sin[140]	Sin[160]	Sin[180]	Si
Sin[21]	Sin[42]	Sin[63]	Sin[84]	Sin[105]	Sin[126]	Sin[147]	Sin[168]	Sin[189]	Si
Sin[22]	Sin[44]	Sin[66]	Sin[88]	Sin[110]	Sin[132]	Sin[154]	Sin[176]	Sin[198]	Si
Sin[23]	Sin[46]	Sin[69]	Sin[92]	Sin[115]	Sin[138]	Sin[161]	Sin[184]	Sin[207]	Si
Sin[24]	Sin[48]	Sin[72]	Sin[96]	Sin[120]	Sin[144]	Sin[168]	Sin[192]	Sin[216]	Si
Sin[25]	Sin[50]	Sin[75]	Sin[100]	Sin[125]	Sin[150]	Sin[175]	Sin[200]	Sin[225]	Si

```
$Aborted
```

As discussed in class, symbolic computations take much longer to perform. The above calculation may take a very long time to perform. In such a case, select the “Evaluation” menu and “Abort Evaluation.”

Let’s run the above again, but this time using numerical values.

```
a = N[Table[Table[Sin[k x], {k, 1, 25}], {x, 1, 25}]];
MatrixForm[a]
RowReduce[a] // MatrixForm
```

0.841471	0.909297	0.14112	−0.756802	−0.958924	−0.279415	0.656987	
0.909297	−0.756802	−0.279415	0.989358	−0.544021	−0.536573	0.990607	-
0.14112	−0.279415	0.412118	−0.536573	0.650288	−0.750987	0.836656	-
−0.756802	0.989358	−0.536573	−0.287903	0.912945	−0.905578	0.270906	
−0.958924	−0.544021	0.650288	0.912945	−0.132352	−0.988032	−0.428183	
−0.279415	−0.536573	−0.750987	−0.905578	−0.988032	−0.991779	−0.916522	-
0.656987	0.990607	0.836656	0.270906	−0.428183	−0.916522	−0.953753	-
0.989358	−0.287903	−0.905578	0.551427	0.745113	−0.768255	−0.521551	
0.412118	−0.750987	0.956376	−0.991779	0.850904	−0.558789	0.167356	
−0.544021	0.912945	−0.988032	0.745113	−0.262375	−0.304811	0.773891	-
−0.99999	−0.00885131	0.999912	0.0177019	−0.999755	−0.0265512	0.99952	0
−0.536573	−0.905578	−0.991779	−0.768255	−0.304811	0.253823	0.73319	
0.420167	0.762558	0.963795	0.986628	0.826829	0.513978	0.105988	-
0.990607	0.270906	−0.916522	−0.521551	0.773891	0.73319	−0.573382	-
0.650288	−0.988032	0.850904	−0.304811	−0.387782	0.893997	−0.970535	
−0.287903	0.551427	−0.768255	0.920026	−0.993889	0.983588	−0.889996	
−0.961397	0.529083	0.670229	−0.897928	−0.176076	0.994827	−0.371404	-
−0.750987	−0.991779	−0.558789	0.253823	0.893997	0.926819	0.329991	-
0.149877	0.296369	0.436165	0.566108	0.683262	0.78498	0.868966	
0.912945	0.745113	−0.304811	−0.993889	−0.506366	0.580611	0.98024	
0.836656	−0.916522	0.167356	0.73319	−0.970535	0.329991	0.609044	-
−0.00885131	0.0177019	−0.0265512	0.0353983	−0.0442427	0.0530836	−0.0619203	0
−0.84622	0.901788	−0.114785	−0.779466	0.945435	−0.228052	−0.702408	
−0.905578	−0.768255	0.253823	0.983588	0.580611	−0.491022	−0.997173	-
−0.132352	−0.262375	−0.387782	−0.506366	−0.61604	−0.714876	−0.801135	-

1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

If we had evaluated at even more test points, it would not have hurt. It would just give us redundant information.

```
a = N[Table[Table[Sin[k x], {k, 1, 25}], {x, 1, 35}]];
MatrixForm[a]
RowReduce[a] // MatrixForm
```

0.841471	0.909297	0.14112	-0.756802	-0.958924	-0.279415	0.656987	
0.909297	-0.756802	-0.279415	0.989358	-0.544021	-0.536573	0.990607	-
0.14112	-0.279415	0.412118	-0.536573	0.650288	-0.750987	0.836656	-
-0.756802	0.989358	-0.536573	-0.287903	0.912945	-0.905578	0.270906	
-0.958924	-0.544021	0.650288	0.912945	-0.132352	-0.988032	-0.428183	
-0.279415	-0.536573	-0.750987	-0.905578	-0.988032	-0.991779	-0.916522	-
0.656987	0.990607	0.836656	0.270906	-0.428183	-0.916522	-0.953753	-
0.989358	-0.287903	-0.905578	0.551427	0.745113	-0.768255	-0.521551	
0.412118	-0.750987	0.956376	-0.991779	0.850904	-0.558789	0.167356	
-0.544021	0.912945	-0.988032	0.745113	-0.262375	-0.304811	0.773891	-
-0.99999	-0.00885131	0.999912	0.0177019	-0.999755	-0.0265512	0.99952	0
-0.536573	-0.905578	-0.991779	-0.768255	-0.304811	0.253823	0.73319	
0.420167	0.762558	0.963795	0.986628	0.826829	0.513978	0.105988	-
0.990607	0.270906	-0.916522	-0.521551	0.773891	0.73319	-0.573382	-
0.650288	-0.988032	0.850904	-0.304811	-0.387782	0.893997	-0.970535	
-0.287903	0.551427	-0.768255	0.920026	-0.993889	0.983588	-0.889996	
-0.961397	0.529083	0.670229	-0.897928	-0.176076	0.994827	-0.371404	-
-0.750987	-0.991779	-0.558789	0.253823	0.893997	0.926819	0.329991	-
0.149877	0.296369	0.436165	0.566108	0.683262	0.78498	0.868966	
0.912945	0.745113	-0.304811	-0.993889	-0.506366	0.580611	0.98024	
0.836656	-0.916522	0.167356	0.73319	-0.970535	0.329991	0.609044	-
-0.00885131	0.0177019	-0.0265512	0.0353983	-0.0442427	0.0530836	-0.0619203	0
-0.84622	0.901788	-0.114785	-0.779466	0.945435	-0.228052	-0.702408	
-0.905578	-0.768255	0.253823	0.983588	0.580611	-0.491022	-0.997173	-
-0.132352	-0.262375	-0.387782	-0.506366	-0.61604	-0.714876	-0.801135	-
0.762558	0.986628	0.513978	-0.321622	-0.930106	-0.881785	-0.210781	
0.956376	-0.558789	-0.629888	0.926819	0.0883687	-0.97845	0.483318	
0.270906	-0.521551	0.73319	-0.889996	0.98024	-0.997173	0.93953	-
-0.663634	0.992873	-0.821818	0.236661	0.467745	-0.936462	0.93331	-
-0.988032	-0.304811	0.893997	0.580611	-0.714876	-0.801153	0.467719	
-0.404038	-0.739181	-0.948282	-0.995687	-0.873312	-0.602024	-0.228082	
0.551427	0.920026	0.983588	0.721038	0.219425	-0.354938	-0.811621	-
0.999912	-0.0265512	-0.999207	0.0530836	0.997797	-0.0795786	-0.995684	
0.529083	-0.897928	0.994827	-0.790433	0.346649	0.20212	-0.689676	
-0.428183	0.773891	-0.970535	0.98024	-0.801135	0.467719	-0.0442126	-

1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Question: are the functions $\{\sin(\pi x), \sin(2\pi x), \sin(3\pi x), \ldots, \sin(10\pi x)\}$ linearly independent?

If we only evaluate at integers, we will not be able to determine the answer.

```
a = Table[Table[Sin[k π x], {k, 1, 10}], {x, 1, 10}];
MatrixForm[a]
RowReduce[a] // MatrixForm
```

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

In this situation, we should try using a different step size.

```
a = Table[Table[Sin[k π x], {k, 1, 10}], {x, 1, 2, .1}];
MatrixForm[a]
RowReduce[a] // MatrixForm
```

1.22465×10^{-16}	-2.44929×10^{-16}	3.67394×10^{-16}	-4.89859×10^{-16}	6.12323×10^{-16}	-7.34
-0.309017	0.587785	-0.809017	0.951057	-1.	0.
-0.587785	0.951057	-0.951057	0.587785	-7.34788×10^{-16}	-0
-0.809017	0.951057	-0.309017	-0.587785	1.	-0
-0.951057	0.587785	0.587785	-0.951057	8.57253×10^{-16}	0.
-1.	3.67394×10^{-16}	1.	-7.34788×10^{-16}	-1.	1.10
-0.951057	-0.587785	0.587785	0.951057	-9.79717×10^{-16}	-0
-0.809017	-0.951057	-0.309017	0.587785	1.	0.
-0.587785	-0.951057	-0.951057	-0.587785	1.10218×10^{-15}	0.
-0.309017	-0.587785	-0.809017	-0.951057	-1.	-0
-2.44929×10^{-16}	-4.89859×10^{-16}	-7.34788×10^{-16}	-9.79717×10^{-16}	-1.22465×10^{-15}	-1.46

1	0.	0.	0.	0.	0.	0.	0.	0.	-5.81895×10^{-17}
0	1	0.	0.	0.	0.	0.	0.	0.	-3.97912×10^{-17}
0	0	1	0.	0.	0.	0.	0.	0.	-1.87197×10^{-16}
0	0	0	1	0.	0.	0.	0.	0.	-8.89758×10^{-17}
0	0	0	0	1	0.	0.	0.	0.	-3.92011×10^{-15}
0	0	0	0	0	1	0.	0.	0.	-1.68558×10^{-16}
0	0	0	0	0	0	1	0.	0.	-7.21051×10^{-16}
0	0	0	0	0	0	0	1	0.	-3.76908×10^{-16}
0	0	0	0	0	0	0	0	1	-2.31963×10^{-15}
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Whoops, the above does not yet show linear independence. Let’s evaluate at a few more values of x. This will allow us to conclude the functions are linearly independent.

```
RowReduce[
  Table[Table[Sin[k π x], {k, 1, 10}], {x, 1, 2, .05}] ] // MatrixForm
```

1	0.	0.	0.	0.	0.	0.	0.	0.	0.
0	1	0.	0.	0.	0.	0.	0.	0.	0.
0	0	1	0.	0.	0.	0.	0.	0.	0.
0	0	0	1	0.	0.	0.	0.	0.	0.
0	0	0	0	1	0.	0.	0.	0.	0.
0	0	0	0	0	1	0.	0.	0.	0.
0	0	0	0	0	0	1	0.	0.	0.
0	0	0	0	0	0	0	1	0.	0.
0	0	0	0	0	0	0	0	1	0.
0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0