MATH 616 HOMEWORK
DUE 4/16/18

Let $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z} = \{0,1\}$ denote the field with the two elements.

(1) Consider the linear map $\mathbb{F}_2^3 \to \mathbb{F}_2^6$ given by

$$(x_1, x_2, x_3) \quad \mapsto \quad (x_1, x_2, x_3, x_2 + x_3, x_1 + x_3, x_1 + x_2).$$

    (a) Determine the image of $(1, 0, 1)$.
    (b) Write the linear map using matrix multiplication.
    (c) Verify (by a very a general abstract argument) that the image of this linear map is a vector subspace $C \subseteq \mathbb{F}_2^6$.
    (d) Write a basis for C, and calculate $\dim C$.
    (e) How many elements will be in the vector space C?

(2) Consider the subset

$$C = \{(y_1, y_2, y_3) \in \mathbb{F}_2^3 \mid y_1 + y_2 + y_3 = 0\} \subseteq \mathbb{F}_2^3.$$

    (a) Write C as the kernel of a linear map. Based on this, what is $\dim C$?
    (b) Write a basis for C.