MTH 6: CONCEPTS OF DERIVATIVES

(Technical note: in the following assume $f(x)$ is a continuous function whose derivative has only a finite number of discontinuities.)

- The derivative of f, with respect to x, is another function and is denoted $f'(x)$ or $\frac{df}{dx}$. Often, we write $y = f(x)$ and denote the derivative by $\frac{dy}{dx}$.
- The derivative is *defined* as the instantaneous rate of change.
- *Geometrically*, $f'(a)$ is the slope of the tangent line to the graph of $f(x)$ at $x = a$. In general, the slope of a line is $\frac{\Delta y}{\Delta x}$, which is ratio between the change in y-values and the change in x-values (hence the notation $f'(x) = \frac{dy}{dx}$).
- It is often convenient to use other variable names. If we think of a quantity y as depending on x, then we can discuss the instantaneous rate of change $\frac{dy}{dx}$. Similarly, we could think of y as depending on t and analyze how y changes when t changes. This leads to the rate of change $\frac{dy}{dt}$, which we call "the derivative of y with respect to t." Example: If a company's daily profit P is changing, and t is measured in days, then $\frac{dP}{dt}$ corresponds to the daily change in profit.
-

$f(x)$	$f'(x)$	$f''(x)$
f is *increasing* at $x = a$	$f'(a) > 0$	
f is *decreasing* at $x = a$	$f'(a) < 0$	
f is *concave up* at $x = a$	f' increasing at $x = a$	$f''(a) > 0$
f is *concave down* at $x = a$	f' decreasing at $x = a$	$f''(a) < 0$

- The *maximum and minimum y-values* (i.e. where the graph is highest and where the graph is lowest) of a continuous function $y = f(x)$ will occur at critical points or endpoints. Critical points are the x-values where $f'(x) = 0$ or undefined.
- To determine if a critical point is a local (=relative) max/min, create a sign chart for $f'(x)$. This tells you where $f(x)$ is increasing and where it is decreasing.
- To find global (=absolute) max/min on a closed interval $[a, b]$, simply plug all the critical points and endpoints into f and compare.
- The idea of a sign chart is the following. Suppose you want to find where $g(x) > 0$ and where $g(x) < 0$. You first find the x where $g(x) = 0$ or is discontinuous. These are the only x-values where g can change $\pm$ signs. You then check whether $g(x)$ is positive or negative *in between* each of the numbers you just found.

On the extra credit quiz for Wednesday 4/23, the questions will be a combination of the following:

- multiple choice or fill in the blank questions about basic concepts,
- you are given a table of values for f, f', f'' and have to answer questions,
- you are given the graph of either f, f', or f'' and have to answer questions.